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AN EXPANSION FOR COULOMB WAVE FUNCTIONS

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Abstract: An expansion for the regular radial Coulomb wave functions, as suggested by an iteration procedure on the wave equation, is developed and found to be more rapidly convergent than the customary expansions. Few terms suffice to give good accuracy for radial distances less than, or of the order of, the position of the classical turning point. This is true for all values of the orbital angular momentum no matter how high. No similar expansion has yet been developed for the irregular Coulomb wave functions.

1. Introduction

There is by now quite an impressive amount of literature (ref.[1a–d]) and others) dealing with Coulomb wave functions (hereafter called CWF-s). As is well known, these functions are expressible in terms of the confluent hypergeometric functions, and fairly extensive tabulations of them are now available [2]).

For some applications, however, for instance those which involve subsequent operations on CWF-s, it would still be very desirable to be able to obtain a simple and tractable form for these functions.

It is our aim in this paper to present briefly an expansion for the regular CWF-s, which is of a very simple nature and yet which converges much more rapidly than the usual expansions. For radial distances less than or about the position of the classical turning point only a few terms need be taken (however high the orbital angular momentum), thus yielding a simple representation of the CWF at least in this range.

The corresponding expansion for the irregular CWF-s has not, as yet, been obtained except in the simplest case of the irregular Bessel functions, i.e. for the zero value of the Coulomb parameter η.

† Also supported by the Nuclear Research Foundation within the University of Sydney.

2. Regular CWF

The general radial CWF-s, denoted by $\bar{F}_L(\varrho;\eta)$, are defined [1a]) as solutions of the radial Coulomb wave equation

$$\left[\mathscr{D}^2 + 1 - \frac{2\eta}{\varrho} - \frac{L(L+1)}{\varrho^2}\right]\bar{F}_L(\varrho;\eta) = 0, \tag{2.1}$$

where

$$\mathscr{D} \equiv d/d\varrho$$

and the symbols are the customary ones, viz.,

$$\varrho = kr = (2\pi \times \text{wave number}) \times (\text{radial distance}),$$

$\eta = ZZ'e^2/\hbar v$, v being the relative velocity of the two particles, and L is the orbital angular momentum quantum number.

In the present paper we shall confine our discussion almost exclusively to the study of regular solutions of (2.1) which we shall write in the usual way [1a, c]) in the form

$$F_L(\varrho;\eta) = C_L(\eta)\varrho^{L+1}\,\Phi_L(\varrho;\eta) \tag{2.2}$$

where

$$C_L(\eta) = \frac{2^L}{(2L+1)!}\left\{(L^2+\eta^2)\left((L-1)^2+\eta^2\right)\ldots(1^2+\eta^2)\frac{2\pi\eta}{e^{2\pi\eta}-1}\right\}^{\frac{1}{2}} \tag{2.3}$$

and

$$\Phi_L(0;\eta) = 1. \tag{2.4}$$

On substituting (2.2) into (2.1), we immediately obtain

$$[\varrho\mathscr{D}^2 + 2(L+1)\mathscr{D} + \varrho - 2\eta]\Phi_L(\varrho;\eta) = 0. \tag{2.5}$$

It is evident from an examination of tabulations of regular CWF-s that the $\Phi_L(\varrho;\eta)$ are smooth and slowly varying functions of ϱ. For not too large values of ϱ, and particularly for high values of L, it should therefore be permissible to neglect, in a first approximation, the second order term $\varrho\,\mathscr{D}^2\Phi_L$ of (2.5) against the remaining terms.

On solving the resulting first order differential equation for $\Phi_L^{(1)}(\varrho;\eta)$, the first approximation to $\Phi_L(\varrho;\eta)$, we obtain

$$\Phi_L^{(1)}(\varrho;\eta) = \exp\left[\frac{\eta\varrho}{L+1} - \frac{\varrho^2}{4(L+1)}\right]. \tag{2.6}$$

In section 3 we shall discuss the accuracy of this function. In any event it can be improved quite efficiently if used as the starting step for an iteration procedure which we shall now describe.

We set up the successive approximation scheme as follows:

$$\varrho\mathscr{D}^2\Phi_L^{(n-1)}(\varrho;\eta)+[2(L+1)\mathscr{D}+\varrho-2\eta]\Phi_L^{(n)}(\varrho;\eta)=0, \quad (2.7)$$

where $\Phi_L^{(n)}(\varrho;\eta)$ is the n^{th} approximation ($n=1, 2, \ldots$) to $\Phi_L(\varrho;\eta)$, and $\Phi_L^{(0)}(\varrho;\eta)$ is chosen to be unity, by definition.

It is then clearly seen that, in general,

$$\Phi_L^{(n)}(\varrho;\eta)=W_L^{[4(n-1)]}(\varrho;\eta)\exp\left[\frac{\eta\varrho}{L+1}-\frac{\varrho^2}{4(L+1)}\right], \quad (2.8)$$

where the $W_L^{[m]}(\varrho;\eta)$ are m^{th} degree polynomials in ϱ; the first two W_L-s are

$$W_L^{(0)}(\varrho;\eta)=1,$$

$$W_L^{(4)}(\varrho;\eta)=1+\frac{\varrho^2}{4(L+1)^2}\left(\frac{1}{2}-\frac{\eta^2}{L+1}\right)+\frac{\eta\varrho^3}{6(L+1)^3}-\frac{\varrho^4}{32(L+1)^3}.$$

The formula (2.8) suggests the possibility of an expansion

$$\Phi_L(\varrho;\eta)=\left(\sum_{\mu=0}^{\infty}a_{L,\mu}(\eta)\varrho^\mu\right)\exp\left[\frac{\eta\varrho}{L+1}-\frac{\varrho^2}{4(L+1)}\right] \quad (2.9)$$

for the exact solution of (2.5). It is apparent that (2.9) will be valid for all finite values of ϱ since both $\Phi_L(\varrho;\eta)$ and the $\exp[(\eta\varrho/(L+1))-(\varrho^2/4(L+1))]$ have infinite radii of convergence and, furthermore, the exponential function has no zeroes for finite values of its argument. It thus follows that the series

$$\sum_{\mu=0}^{\infty}a_{L,\mu}(\eta)\varrho^\mu \quad (2.10)$$

must have an infinite radius of convergence.

Because of the initial requirement (2.4), we must have

$$a_{L,0}(\eta)=1. \quad (2.11)$$

On substituting (2.9) into (2.5), we therefore find for the first four $a_{L,\mu}$:

$$\left.\begin{aligned} a_{L,0}&=1, \qquad a_{L,2}=\frac{L+1-2\eta^2}{4(L+1)^2(2L+3)},\\ a_{L,1}&=0, \qquad a_{L,3}=\frac{\eta[(L+1)^2+\eta^2]}{3(L+1)^3(L+2)(2L+3)}. \end{aligned}\right\} \quad (2.12)$$

In addition we obtain the general recurrence relationship

$$(\nu+1)(\nu+2(L+1))a_{L,\nu+1}+\frac{1}{L+1}\left\{\eta\left[2\nu\, a_{L,\nu}-\frac{a_{L,\nu-2}}{L+1}\right]+\right.$$
$$\left.+\frac{2\eta^2-(2\nu-1)(L+1)}{2(L+1)}a_{L,\nu-1}+\frac{a_{L,\nu-3}}{4(L+1)}\right\}=0, \quad (2.13)$$

for $\nu \geqq 3$, from which, e.g.,

$$a_{L,4}=-\frac{(4L+1)(L+1)^2(L+2)+4\eta^2[(7-\eta^2)L+10-2\eta^2]}{32(L+1)^4(L+2)(2L+3)(2L+5)}.$$

In the next section we investigate the convergence-rate of expansion (2.10) and compare it with the usual series expansion

$$\Phi_L(\varrho;\eta)=\sum_{\varkappa=0}^{\infty}A_{L,\varkappa}(\eta)\varrho^{\varkappa}. \quad (2.14)$$

Here the coefficients $A_{L,\varkappa}$ satisfy the recurrence relation

$$A_{L,\varkappa}=\frac{2\eta A_{L,\varkappa-1}-A_{L,\varkappa-2}}{\varkappa(2L+\varkappa+1)} \quad (2.15)$$

so that

$$\left.\begin{aligned} &A_{L,0}=1, \qquad A_{L,2}=\left\{\frac{2\eta^2}{L+1}-1\right\}\frac{1}{2(2L+3)},\\ &A_{L,1}=0, \qquad A_{L,4}=\frac{\eta[2\eta^2-3L-4]}{6(L+1)(L+2)(2L+3)}, \text{ etc.}\end{aligned}\right\} \quad (2.16)$$

We shall see in section 3 that the series (2.10) is much more rapidly convergent than (2.14).

We shall close this section with a mention of the irregular solutions of (2.1), i.e. the functions

$$G_L(\varrho;\eta)=D_L(\eta)\varrho^{-L}\Theta_L(\varrho;\eta) \quad (2.17)$$

with

$$D_L=\frac{1}{(2L+1)C_L},$$

and where the function Θ_L may be expressed in the form

$$\Theta_L(\varrho;\eta)=\Psi_L(\varrho;\eta)+\varrho^{2L-1}(p_L(\eta)\ln 2\varrho+q_L(\eta))\Phi_L(\varrho;\eta). \quad (2.18)$$

Here Ψ_L is a well-behaved function of ϱ such that

$$\Psi_L(0;\eta)=1, \quad (2.19)$$

and $p_L(\eta)$, $q_L(\eta)$ are functions of L, η such that

$$p_L(0)=q_L(0)=0. \quad (2.20)$$

It is evident that we cannot apply to $\Theta_L(\varrho;\eta)$ itself (for a general η) any procedure analogous to the one we have employed successfully in the case of $\Phi_L(\varrho;\eta)$. This is so since the additive term containing a logarithmic factor ln 2ϱ *cannot* be represented by a power series in ϱ. We may, however, still use a corresponding treatment for the function $\Psi_L(\varrho;\eta)$ alone. We hope to be able to deal fully with this problem in a later publication.

We note, however, from (2.18) with (2.20) that in the case of $\eta = 0$

$$\Theta_L(\varrho;0) = \Psi_L(\varrho;0)$$

is a *regular* function and satisfies the equation

$$(\varrho\mathscr{D}^2 - 2L\,\mathscr{D} + \varrho)\,\Theta_L(\varrho;0) = 0. \tag{2.21}$$

Comparing (2.21) with (2.5) and then (2.17) with (2.2), and remembering that both F_L and G_L are solutions of (2.1), we may conclude that, at least formally, we can obtain $\Theta_L(\varrho;0)$ from $\Phi_L(\varrho;0)$ on replacing everywhere $(L+1)$ by $(-L)$. An exact calculation similar to the one made in the case of Φ_L does, in fact, lead us to the same result. It gives

$$\Theta_L(\varrho;0) = \left(\sum_{\mu=0}^{\infty} b_{L,\mu}\varrho^{\mu}\right)\exp\left(\frac{\varrho^2}{4L}\right), \tag{2.22}$$

for $L \neq 0$, where

$$\begin{aligned} b_{L,0} &= 1, \qquad b_{L,2} = \frac{1}{4L(2L-1)}, \\ b_{L,1} &= 0, \qquad b_{L,3} = 0, \end{aligned} \tag{2.23}$$

with the general recurrence relation

$$(\nu+2)(\nu-2L+1)b_{L,\nu+2} + \frac{1}{2L}\left\{(2\nu+1)b_{L,\nu} + \frac{b_{L,\nu-2}}{2L}\right\} = 0 \tag{2.24}$$

for $\nu = 2, 4, \ldots$, all the b's for odd ν vanishing identically.

The convergence of the series

$$\sum_{\mu=0}^{\infty} b_{L,\mu}\varrho^{\mu}$$

will be essentially identical to the corresponding $\eta = 0$ case of (2.10) which is considered in section 3; in analogy with (2.10) therefore we know that the above series is quite rapidly convergent for distances of order of or less than the classical turning point. Of course, eq. (2.22) and, when $\eta = 0$, also eq. (2.9) are essentially

representations of the well known spherical Bessel functions which are not only amply tabulated, but are made up of elementary trigonometrical functions. Even in this case, however, eq. (2.9) and (2.22) may find application, for example in coding problems for an electronic computer. For high L values it may be very convenient to use a rapidly convergent series in which no great cancellation occurs, rather than to code the machine to calculate spherical Bessel functions from their exact expressions, which for distances inside the classical turning point involve cancellations to a very high degree.

3. Rate of Convergence of Series

The fact that the series (2.9) is, in general, quite rapidly convergent at least for distances less than or of order of the classical

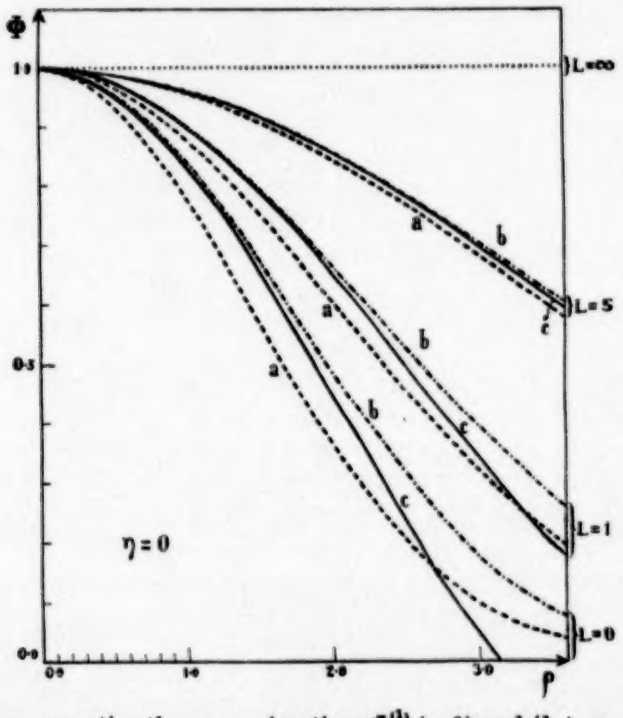

Fig. 1. Curves representing the approximations $\Phi_L^{(1)}(\varrho; 0)$ and $(1 + a_{L,2}(0)\varrho^2)\Phi_L^{(1)}(\varrho; 0)$ and marked respectively as **a** and **b**, are plotted together with the exact function $\Phi_L(\varrho; 0)$ — curve **c** — for values of $L = 0$, 1, and 5. The line $\Phi_\infty^{(1)}(\varrho; 0) \equiv \Phi_\infty(\varrho; 0) = 1$ has been *dotted* in for comparison. Note the fast improvement of the approximations with increasing L.

turning point is illustrated by the curves of Figs. 1, 2, and 3. In Figs. 1 and 2 we have plotted the results yielded by (2.9) when only the first term of the series is taken, and also when a few additional terms are included. The resulting curves are compared with those

corresponding to the exact function Φ_L. Fig. 1 corresponds to the case $\eta = 0$, and Fig. 2 to the case $\eta = 2$, and in each case we have considered the three values 0, 1, and 5 for L.

In Fig. 3 the exact function Φ_L is compared to the simple function $\Phi_L^{(1)}$ yielded by the first term of (2.9) for the cases $L = 5, \eta = 0, 1, 2$.

From the illustrative examples of Figs. 1 and 2 we see that for values of ϱ corresponding to distances less than the classical turning point, a quite rapid convergence is achieved. The reason for this, of

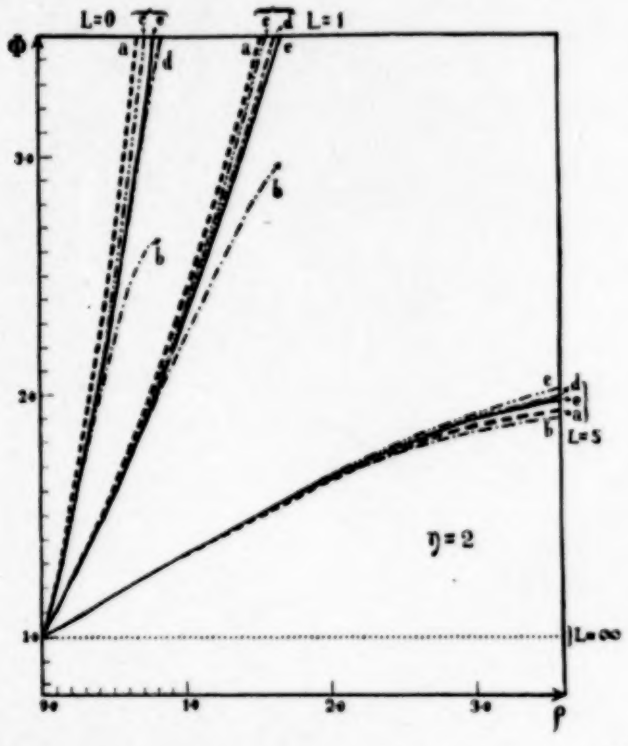

Fig. 2. Curves representing the approximations $\Phi_L^{(1)}(\varrho; 2)$ and $(1+\sum_{\nu=1}^{r+2} a_{L,\nu}(2)\varrho^\nu)\Phi_L^{(1)}(\varrho; 2)$ with $r = 0$, 1, and 2 and marked consecutively **a** to **d**, are plotted together with the exact function $\Phi_L(\varrho; 2)$ — curve **e** — for values of $L = 0$, 1, and 5. The line $\Phi_\infty^{(1)}(\varrho; 2) \equiv \Phi_\infty(\varrho; 2) = 1$ has been *dotted* in for comparison. An arrow at the end-point of a curve (when within the frame of the graph) indicates that this curve is discontinued since it starts to deviate violently at this point from the corresponding exact function.

course, is that the multiplying factor $\Phi_L^{(1)}$ of (2.9) is itself a quite reasonable first approximation (see also Fig. 3) so that the series of (2.9) plays the role merely of a correction factor, at least for the distances discussed.

The reasons why the multiplying factor $\Phi_L^{(1)}$ is a good first approximation may be readily seen. We note that, apart from the trivial

relation

$$\Phi_L^{(1)}(0;\eta) = \Phi_L(0;\eta),$$

we also have (actually as an immediate consequence of our method of deriving (2.6)),

$$\left[\frac{\partial}{\partial\varrho}\Phi_L^{(1)}(\varrho;\eta)\right]_{\varrho=0} = \left[\frac{\partial}{\partial\varrho}\Phi_L(\varrho;\eta)\right]_{\varrho=0}.$$

Also, $\Phi_L^{(1)}(\varrho;\eta)$ is a Gaussian in ϱ with its maximum

$$\lambda(L,\eta) = \exp\left(\frac{\eta^2}{L+1)}\right)$$

situated at

$$\bar{\varrho}(\eta) = 2\eta$$

and with a width parameter

$$\sigma(L) = (2(L+1))^{\frac{1}{2}}.$$

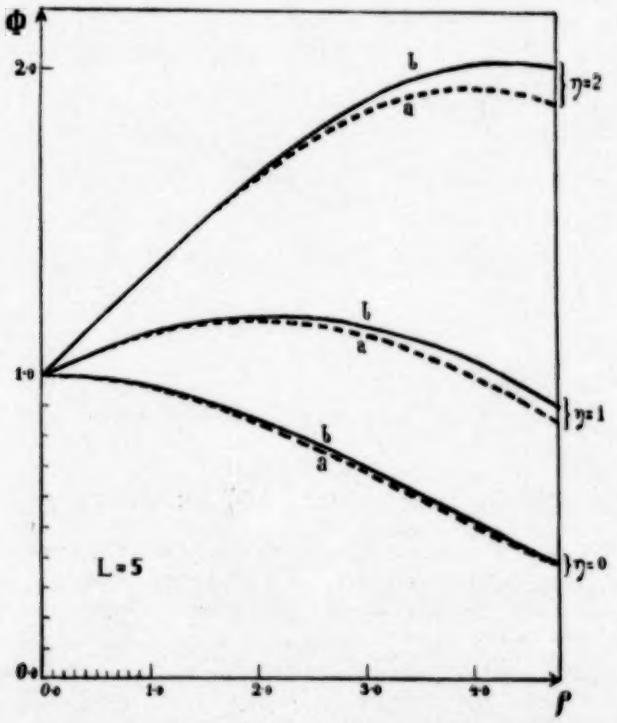

Fig. 3. Curves representing $\Phi_5^{(1)}(\varrho;\eta)$ and $\Phi_5(\varrho;\eta)$ and marked respectively as **a** and **b**, are plotted for values of $\eta = 0$, 1, and 2. Note how the position and height of peaks vary with η.

All the qualitative consequences of these features are true for the half-width, position, and variation with L and η of the first maximum of the exact function $\Phi_L(\varrho;\eta)$.

As may easily be verified, a very large number of terms of the usual series (2.14) must be summed to give accuracy comparable with the first one or two terms of our expression (2.9), particularly for large values of L.

We are greatly indebted to Professor H. Messel for the excellent research facilities made available to us. In addition, one of us (J.R.S.) is grateful to the University of Sydney Research Committee for the grant of a Research Studentship during the tenure of which this work was carried out.

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FISSION OF URANIUM WITH 170 MeV PROTONS

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Abstract: The formation cross section of 22 products formed in fission and spallation of natural uranium with 170 MeV protons have been measured. The integrated fission cross section is determined to (1.9 ± 0.3) barn. Improved half-lives are reported for ^{89}Sr and ^{66}Ni, respectively (51 ± 1)d and (54.8 ± 0.3)h. The energy of the β-particles from ^{66}Ni is determined to 0.15 MeV.

Early in 1954 a study was started of the yield distribution for fission and spallation products formed in bombardment of natural uranium with protons accelerated to 170 MeV in the synchrocyclotron of the Gustaf Werner Institute for Nuclear Chemistry.

The present paper summarizes the main experimental results obtained on the mass distribution. This paper will later be followed by a more detailed one in which will be included also the results from the investigation now in progress of the charge distribution.

The yields of the fission and spallation products are measured by the techniques of radiochemical separations followed by the identification of the nucleides of interest through radioactivity measurements.

The standard radiochemical procedures [1,2,3]) were in general applicable for this study and only minor modifications were sometimes introduced in order to obtain samples suitable for counting of absolute disintegration rates. In a few cases, however, the recommended separation methods resulted in radiochemically impure samples. The impurities were ascribed to the fact that products from symmetrical or near symmetrical division are preferentially formed in high energy fission, while thermal and low energy fission also gives the same products but with much lower yields in the

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valley region relative to the maximum yield (a factor much less than 10^{-2}). Only one such case, however, will be reported here.

The procedure given by Glendenin [4]) for separation of strontium from thermal fission mixture and frequently used in high energy fission studies was found to result in samples containing a 55h impurity with an approximately 3h daughter growing in. This impurity was shown to be due to the presence in the sample itself of 43d ^{115m}Cd and 53h ^{115}Cd, the latter giving rise to 4.5h ^{115m}In. As 51d ^{89}Sr frequently is used as an internal standard in fission yield measurements, the presence of an activity with nearly the same half-life may result in serious errors. Additional scavenging and precipitation steps were therefore introduced in the procedure, which thus yielded radiochemically pure strontium samples.

Using the improved separation method the half-life of ^{89}Sr was determined to (51 ± 1)d as an average of 17 samples, some of which were followed over four half-lives. Corrections were made for the presence in small amounts of the 19.9y ^{90}Sr in equilibrium with the 64.6h ^{90}Y †. The new half-life is lower than the 53d reported by Novey *et al.* [6]), but corroborated by the half-life of (50.4 ± 0.5)d recently published by Hermann [7]).

An improved half-life was also measured for ^{66}Ni: (54.8 ± 0.3)h as an average of 3 samples followed for about ten half-lives. The new value is somewhat lower than the 56h reported by Goeckermann and Perlman [8]) and by Hopkins [9]), but is considered to be more reliable due to the way the measurements were carried out, e.g. good statistics, long decay and treatment of data. The energy of the β-particles following the decay of ^{66}Ni was determined by Feather analysis to about 0.15 MeV. Nervik and Seaborg [10]) assume 0.30 MeV, but without any details.

The activities of the isolated fission and spallation products were measured by the techniques of absolute β-counting and converted to absolute disintegration rates as described by Pappas [11]).

Using these methods the yields relative to the 51d ^{89}Sr were measured for 21 products in the mass region 65 through 140 and around 228. The nucleides studied and their relative yields are given in Table I. The reported values are averages of two to five independent determinations. The limits listed are based on an estimate of the

† Wiles and Tomlinson [5]) give 27.7 y for the half-life of ^{90}Sr. Using this value instead of 19.9 y has no detectable influence on the half-life of ^{89}Sr as reported here.

errors involved in the measurements themselves and are larger than the mean deviation between single measurements. In the last column of Table I the formation cross sections are given. These were computed after normalizing the yield of the monitor 51d ^{89}Sr, by separating and measuring the ^{24}Na formed from aluminium in the

TABLE I

Fission cross sections

Nucleide	Yield relative to ^{89}Sr	Cross section mb
^{65}Ni	0.026±0.001	0.96±0.06
^{66}Ni	0.028±0.002	1.04±0.09
^{82}Br	0.025±0.004	0.93±1.2
^{83}Br	0.16 ±0.03	5.9 ±1.2
^{89}Sr	1.00	37 ±2
^{90}Sr	0.73 ±0.03	27 ±2
^{99}Mo	1.8 ±0.2	67 ±6
^{103}Ru	1.4 ±0.4	52 ±13
^{105}Ru	1.8 ±0.2	67 ±8
^{106}Ru	1.6 ±0.4	59 ±15
^{110m}Ag	0.05 ±0.02	2 ±1
^{111}Ag	1.71 ±0.13	63 ±5
^{112}Pd	1.6 ±0.4	59 ±15
^{124}Sb	0.07 ±0.01	2.6 ±0.4
^{127}Sb	0.6 ±0.2	22 ±7
^{132}Te	0.29 ±0.02	10.7 ±0.9
^{131}I	0.79 ±0.02	29 ±2
^{135m}Ba	0.22 ±0.03	8.1 ±1 2
^{139}Ba	0.82 ±0.05	30 ±2
^{140}Ba	0.68 ±0.06	25 ±3
^{227}Th	0.04 ±0.02	1.5 ±0.8
^{228}Th	0.04 ±0.1	1.5 ±4

reaction ^{27}Al (p,3pn) ^{24}Na with a cross section of (10.8±0.5) millibarn as reported by Folger and Stevenson [12]). The comparison was done by a procedure introduced by Rudstam [13]) who intimately mixes aluminium powder with the target (uranium trioxide) before irradiation thus ensuring the same irradiation conditions in both targets.

Lindner and Osborne [14]) have measured excitation functions for a few selected nucleides formed in proton induced fission of uranium. For these nucleides the cross sections reported here are in excellent agreement with values taken from the excitation curves of Lindner and Osborne.

A plot of the measured cross sections versus mass number de-

pendence show the single peaked mass distribution characteristic for high energy (>100 MeV) fission. From this curve it can be estimated that, on the average, roughly six nucleons (neutrons) are emitted from the excited target nucleus and/or the excited fission fragments.

Integrating under the mass distribution curve yields a fission cross section of (1.9 ± 0.3) barn, i.e. close to the geometrical cross section of ^{238}U (≈ 2.3 barn). This is much higher than the 0.5 barn reported by Jungerman [15]), but is recently supported by the value of 2 barn independently determined by Folger, Stevenson, and Seaborg [16]) for 340 MeV protons and by Vinogradov *et al.* [17]) for 480 MeV protons.

The authors want to express their gratitudes to Professor Dr. The Svedberg, director of the Gustaf Werner Institute for Nuclear Chemistry, for offering the synchrocyclotron for these studies and for the excellent arrangements made in order to perform and continue this investigation. It is also a pleasure to acknowledge stimulating discussions with Drs. S. G. Rudstam and H. Tyrén. The assistance given us by the cyclotron group under the supervision of Mr. Å. Svanheden is gratefully acknowledged.

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POSSIBLE FINITE NUCLEAR SIZE EFFECT ON THE 279 keV TRANSITION IN Tl^{203}

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Much attention has recently been focused on the deexcitation of the 279 keV state in Tl^{203} produced either by the β decay of Hg^{203} or electron capture of Pb^{203}. The purpose of the present investigation was to settle the question of conversion coefficients and multipole mixing of the 279 keV transition in Tl^{203} in regard to which previous investigations have shown a discrepancy.

As already pointed out by Wapstra and Nijgh [1]) the absolute K conversion coefficients obtained by different investigators fall into two categories, one grouping around the value 0.20 and the other around 0.16. Starting from their value of $\gamma_K = 0.16$ and their value 3.35 for the K/L ratio Wapstra and Nijgh were led to draw the conclusion that the absolute K and L_I conversion coefficients for magnetic dipole radiations, as calculated and tabulated by Rose [2]), had to be corrected by a factor ≈ 0.60. This correction was ascribed to the effect of the structure of the nucleus, neglected in the theoretical calculations. The relative conversion intensities of a pure magnetic dipole transition of energy 238 keV in Bi^{212} were studied by Sokolowski, Edvarson and Siegbahn [3]), who concluded that if the finite size effect is at all pronounced in this region of Z, it should be approximately equal for K, L_I and L_{II} conversion. It is in the light of the above results that the present investigation was undertaken.

The source used was Hg^{203}, produced in the Materials Testing Reactor, Idaho, and mass separated onto a 150 $\mu g/cm^2$ Al foil. The source dimensions were 0.5×10 mm^2 and the thickness estimated to be less than 10 $\mu g/cm^2$. The absolute K conversion

coefficient was determined using an iron yoke double-focusing spectrometer [4]) of the Werner Institute at Uppsala. γ_K was obtained by letting the intensity of the β continuum represent the total decay intensity. The source foil was mounted on a ring-shaped source holder of 4 cm diameter, covered with a ≈ 60 μg/cm^2 formvar foil to minimize scattering. To prevent the mercury from escaping into the vacuum tank the source itself was covered with

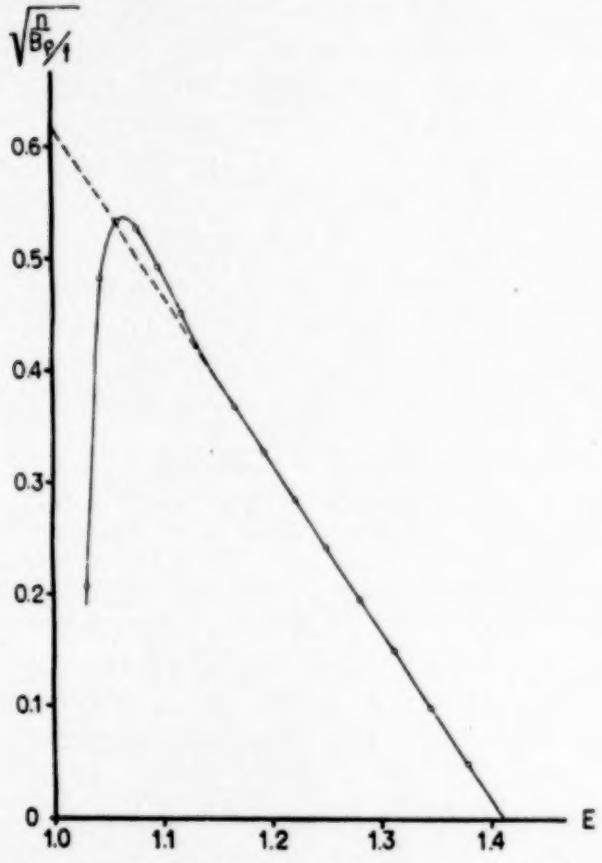

Fig. 1. Fermi plot of the beta spectrum of Hg^{203}.

a formvar foil approximately 30 μg/cm^2 thick. The GM detector had a "rubberhydrochloride" window of thickness 400 μg/cm^2. From the Fermi plot (fig. 1) it is seen that disturbances in the continuum set in at roughly 80 keV. The electron excess above the window absorption region is attributed to scattering. The low energy part of the curve used for the intensity determination was constructed from the extrapolated high energy part of the Fermi plot (fig. 2), and thus the determination of the absolute K conversion coefficient is based on the assumption that the β transition should yield a linear Fermi plot.

To determine the relative intensities of the K, L_I, L_{II} and L_{III} conversion lines an iron-free double-focusing spectrometer [5]) was employed with the resolution set at 0.9 $^0/_{00}$. The intensity values thus obtained, together with the absolute K conversion coefficient, give the absolute L_I, L_{II} and L_{III} coefficients.

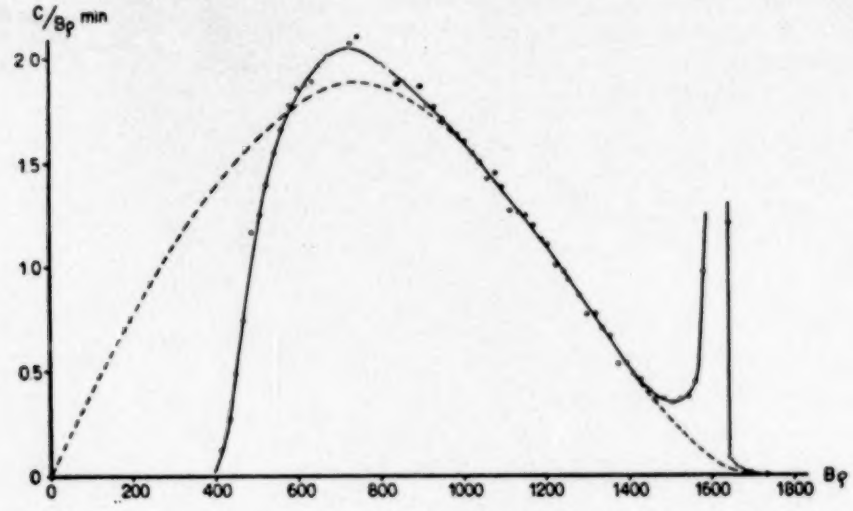

Fig. 2. Continuous beta spectrum of Hg^{203} obtained experimentally and corrected spectrum obtained from Fermi plot in fig. 1.

According to earlier measurements of the K and L conversion coefficients the transition under investigation should be a mixture of M1 and E2 or E1 and M2 radiation. Our value 1.5 for the L_I/L_{II} ratio makes the second possibility rather improbable: according to Rose's theoretical data L_I/L_{II} varies between 0.5 and 10 for M1+E2, but between 4.6 and 7.5 for E1+M2. The following relations hold between the mixing ratio a, the electric quadrupole conversion coefficients α and the magnetic dipole coefficients β:

$$\gamma_i = a\alpha_i + (1-a)\beta_i, \qquad i = \mathrm{K},\ \mathrm{L_I},\ \mathrm{L_{II}},\ \mathrm{L_{III}}, \quad (1)-(4)$$

where γ is the conversion coefficient determined experimentally. Further, we may write

$$\beta_i = X_i \beta_{Ri}, \qquad i = \mathrm{K},\ \mathrm{L_I},\ \mathrm{L_{II}},\ \mathrm{L_{III}}, \qquad (5)$$

where X is the correction factor to be applied to the theoretical values β_R of Rose. No correction is assumed for E2 transitions. From the theoretical point of view [6]) E2 corrections might be justified but are expected to be so small, that their effect would lie within our experimental errors. Experimental data, e.g. for the

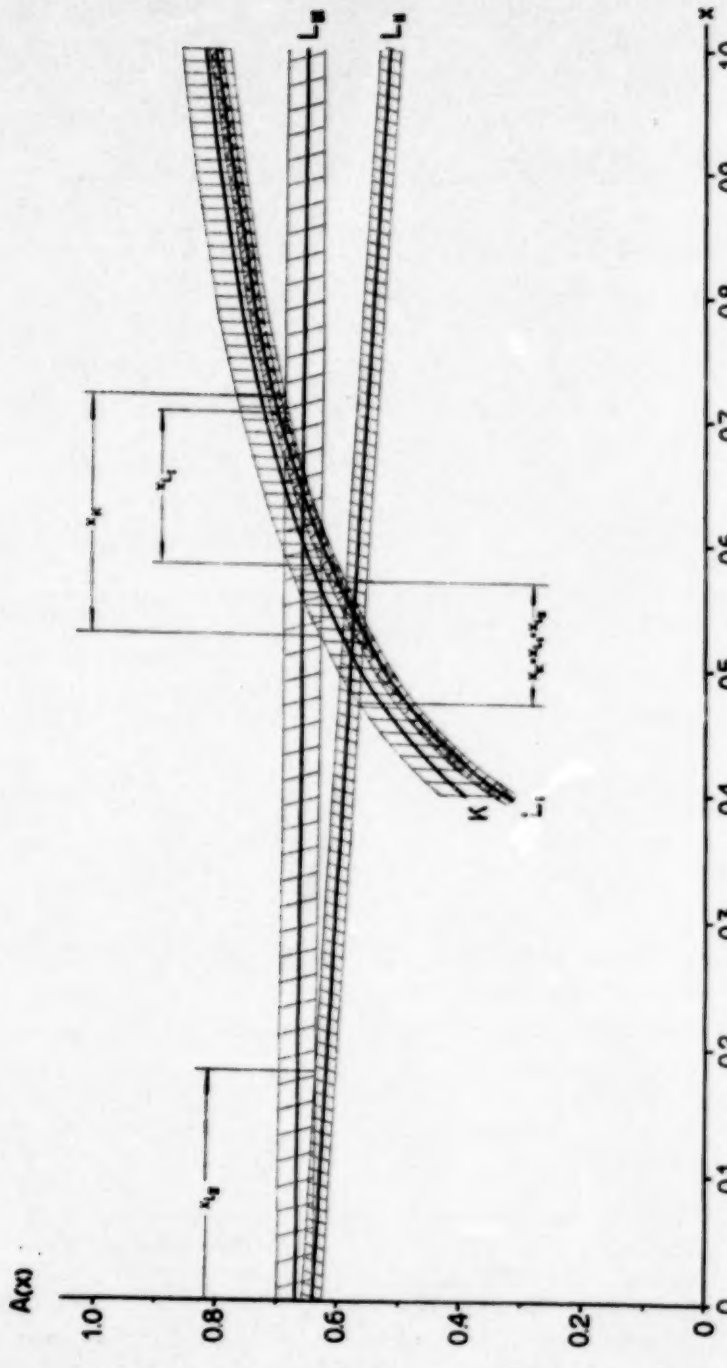

Fig. 3. Graphs of eqs. (1)—(4) showing the mixing ratio as a function of the correction factor X for the K, LI, LII, LIII coefficients. The strips indicate the experimental errors in the relative intensity measurements and the interpolation errors.

411 keV transition following the decay of Au^{198} [7]), further justify our treatment.

To determine a and X_i we may start from two different assumptions. In view of ref. [4]) we may assume that $X_K = X_{L_I} = X_{L_{II}}$. Rewriting equations (1)—(3) in the form

$$A_i(X) = a = \frac{\gamma_i - X\beta_{Ri}}{\alpha_i - X\beta_{Ri}}, \qquad i = K,\ L_I,\ L_{II}, \tag{6}$$

and plotting A as a function of X we get three curves, which *according to the above assumption* should intersect in the same point, the ordinate of which is the desired mixing ratio, the abscissa giving the correction factor $X_K = X_{L_I} = X_{L_{II}}$. Fig. 3 illustrates the procedure. The errors indicated by the strips around the curves

TABLE I

	Theor. E2 conv. coeff. acc. to Rose	Theor. M1 conv. coeff. acc. to Rose	Experim. conv. coeff. I	Experim. conv. coeff. II	Calculated conv. coeff. $a=0.58$ $X_K=X_{L_I}=X_{L_{II}}=0.53$ $X_{L_{III}}=1.0$	Calculated conv. coeff. $a=0.66$ $X_K=X_{L_I}=0.65$ $X_{L_{II}}=0.2$ $X_{L_{III}}=1.0$	a calculated with $X_K=X_{L_I}=X_{L_{II}}=X_{L_{III}}=1.0$	Conv. coeff. calculated with $X_K=X_{L_I}=X_{L_{II}}=X_{L_{III}}=1.0$, $a=0.692$
K	0.0763	0.53	0.159 ±0.004	0.164	0.162	0.167	0.821	0.217
L_I	0.011	0.079	0.0247 ±0.0003†	0.0244	0.0240	0.0248	0.795	0.032
L_{II}	0.0252	0.0070	0.0160 ±0.0002†	0.0160	0.0162	0.0171	0.494	0.0196
L_{III}	0.0126	0.00043	0.0084 +0.0002†	0.0085	0.0075	0.0085	0.657	0.0089

† These errors do not include the error in γ_K, since the latter enters as a systematic error into all four coefficients.

Of the two sets of experimental values the first is based on the measurements with the iron yoke and iron-free double-focusing spectrometers reported in this paper. The second set is based on the Dutch measurements with an iron-free lens spectrometer and the relative intensity measurements of ref [1]).

The theoretical conversion coefficients were obtained by graphical interpolation from Rose's tables for the appropriate Z and energy.

The two sets of calculated coefficients are based on the two independent determinations of X and a, described in the text.

Taking the average of the values in column 7 gives $a = 0.692$. This value is used to obtain the conversion coefficients of column 8, where again no correction is applied to the magnetic dipole coefficients.

are those resulting from the *relative* intensity measurements and the interpolation of α anf β. The error in γ_K would occur as a systematic error in all four curves and is therefore not shown in the figure. Taking this error into account as well we obtain $X_K = X_{L_I} = X_{L_{II}} = 0.53 \pm 0.08$ and the mixing ratio $a = 0.58 \pm 0.04$.

In figure 3 the function $A_{L_{III}}$ is also plotted as a function of X but is seen to be very insensitive to a change in X, and hence not much can be said about $X_{L_{III}}$. However, we may use this fact for an independent determination of a, X_K, X_{L_I} and $X_{L_{II}}$. Whether we assume $X_{L_{III}}$ equal to or less than 1, we obtain $a = 0.66 \pm 0.04$. Starting from this value we get $X_K = 0.65 \pm 0.15$ and $X_{L_I} = 0.67 \pm 0.12$. In the same way we get for $X_{L_{II}}$ a value 0.23 ± 0.43. The latter is in disagreement with the results of ref. [1,3]) and other investigations of pure M1 transitions. We do not wish to exclude the possibility that our experimental errors are somewhat greater than estimated. This possibility is greatest in the case of the L_{III} measurements, which would not affect the results of the preceding paragraph.

Calculations by Sliv *et al.* [8]) indicate a possible theoretical correction factor for the K and L_I magnetic dipole conversion coefficients due to finite nuclear size of 0.65 for $Z = 83$.

We are greatly indebted to Dr Almén of Chalmers Institute of Technology, Göteborg, for undertaking the mass separation of our sample. Further, our thanks are due to Dr Arbman of the Gustav Werner Institute, University of Uppsala, for putting his double focusing iron yoke spectrometer at our disposal.

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EFFECT OF A SHORT-RANGE REPULSIVE INTERACTION IN THE (jj) COUPLING SHELL MODEL

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A much discussed problem of nuclear spectroscopy has been that the level of angular momentum $I=7/2$ occurs in some cases as the most stable level of the $(g_{9/2})^{\pm 3}$ configuration [1]). Calculations of Talmi [2]) and Flowers [3]) show that $I=7/2$ is not expected to be the lowest state with the usual monotonic two-nucleon central potentials, although the level spacing of $I=7/2$ and the lowest state $I=9/2$ is quite small [4]). It may be assumed that this discrepancy indicates the breakdown of the single-particle approximation [5]). I wish to suggest, however, that there exists a possibility of explaining this fact within the framework of the single-particle shell model.

In recent years numerous results in the meson field theory and in nuclear physics indicated the existence of a highly singular short-range repulsive interaction between nuclei [6]). We have studied the consequences of this type of interaction for the level order of nuclear configurations. The calculations were based on a purely phenomenological two-nucleon potential:

$$V_{ik}=-O_{ik}V_0\frac{e^{-(\mu r)^2}}{\mu r}+\alpha V_0\frac{e^{-(2\mu r)^2}}{(2\mu r)^2}. \qquad (1)$$

The operator O_{ik} determines the exchange character of the part of the potential which is attractive in symmetrical states. We have considered the cases

$O_{ik}=1$ (Wigner-force),
$O_{ik}=\frac{1}{2}(1+P_{ik})$ (Serber-force),
$O_{ik}=P_{ik}$ (Majorana-force).

The parameter α is the measure of the strength of the repulsion. The values of α causing a crossover of the levels 7/2 and 9/2 of

$(g_{9/2})^3$ were determined using the method proposed by Talmi [2]). Let us denote the quantity $\nu^{1/2}\mu^{-1}$ by λ, where the parameter ν occurs in the oscillator wave functions in the same manner as in Talmi's work; our results for those values of λ which are of physical interest can be seen in the following table:

$\lambda =$	1	2	3
Wigner	1.21	0.20; 0.67	0.22; 0.39
Serber	1.02	0.15; 1.25	0.06; 0.13
Majorana	0.84	0.40	0.16; 0.41

In some cases there exist two values of α because of the loss of diagonality of the seniority operator [2]). The resulting values of α are quite reasonable; however, the relation of our repulsive potential to others discussed in the literature needs further clarification.

The effect of the repulsion on the states of $(d_{5/2})^3$ has also been studied. It is well known that the anomalous spin of Na^{23} can only be explained by the assumption of a non-singular two-nucleon potential [7]); in the case of a singular potential of the type $\exp(-\mu r)/r$ or $\exp(-\mu^2 r^2)/r$ the range of force μ^{-1} has to be chosen too large [8,2]). Our calculations show that the potential (1) for values of α approximately equal to those found in the case of $(g_{9/2})^3$ yields nearly the same results as the calculations of Kurath [7]) applying the non-singular Gaussian potential.

We have assumed a short range repulsive potential of the Wigner type adopting the qualitative features of the Lévy potential [6]), which contains a spin and isobaric spin independent repulsive core. Calculation shows, however, that for the $(d_{5/2})^3$ configuration a repulsive potential of the same exchange character as the attractive one causes essentially the same effect as the potential discussed here.

Although we do not intend to discuss the various repulsive potentials deduced from the meson field theories or scattering experiments, it may be noted that a repulsion nearly equal to the repulsion used in our calculations acts in a static scalar field as a relativistic effect [9]).

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SCATTERING PROBLEMS IN FIELD THEORY

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Abstract: An approximation procedure which starts from a reformulation of Fredholm's type of solution is given. Applications to S matrix theory, to the theory of the reactance operator, to the determination of adiabatic nuclear potentials and to Heitler's integral equation are given.

It is well-known that the solution of a scattering problem in the S matrix theory is expressed generally by the perturbation series, i.e. by an expansion of the solution in terms of the powers of the coupling constant g. The coefficient of g^n involves the contribution of all n-th order Feynman graphs describing the kind of scattering which is studied.

For large g, the approximation obtained by stopping this series at a given g^n is certainly meaningless. Our purpose is to investigate a method of solution, which may be applied especially to problems of scattering and which is valid for strong and weak coupling. This solution, as shown later, may be deduced from the perturbation series by replacing the coefficient of g^n by a linear combination of all the contributions from the Feynman graphs of order zero to n, every class of graphs having a given weight.

Considering, for simplicity, the integral equation leading to the S matrix theory, we shall point out that this result may be obtained by using Fredholm's method with some rather important modifications. First of all it should be noticed that since the kernel of this integral equation is a q-number, a reformulation of this method is necessary. This was done in a previous paper[1]) (quoted hereafter as A). In the present paper, following the general method of A, we approximate to the exact solution of the S matrix equation by the ratio of two polynomials in g of degree N plus an operator

$\mathbf{R}'$ representing the difference between true and approximate solutions, and we consider the equation satisfied by $\mathbf{R}'$.

The denominator of the former fraction contains, as shown in A, a set of arbitrary c-numbers D_n. If $|\mathrm{I}\rangle$ is the initial state, they will be chosen in such a way that the length of the vector $\mathbf{R}'|\mathrm{I}\rangle$ is small enough and this is the condition which will determine completely the numbers D_n and therefore the solution.

We shall begin in the first section by a rather abstract and formal survey of linear equations (as in A), then the choice of this arbitrary set of numbers D_n will be discussed. Finally, applications to S matrix theory, as well as some other applications to problems of field theory will be given.

1. The Approximation Method

Let us consider the linear equation

$$U = U_0 + \lambda \mathbf{K} U, \tag{1.1a}$$

where $\mathbf{K}$ is a given linear operator. U_0 is a known operator, λ a real number which is not an eigenvalue of the operator $\mathbf{K}$ and U the unknown operator. Equation (1.1a) may be transformed into a vector equation by applying it to a given vector $|0\rangle$ in Hilbert space:

$$X = X_0 + \lambda \mathbf{K} X, \tag{1.1b}$$

with

$$X = U|0\rangle, \quad X_0 = U_0|0\rangle.$$

The resolvent † $\mathbf{R}$ which satisfies the known equation (A 1.4)

$$\mathbf{R} = \mathbf{K} + \lambda \mathbf{K}\mathbf{R} \tag{1.2}$$

allows us to write the solutions of the equations (1.1) in the following form:

$$U = U_0 + \lambda \mathbf{R} U_0, \tag{1.3a}$$

$$X = X_0 + \lambda \mathbf{R} X_0. \tag{1.3b}$$

Since $\mathbf{R}$ is an operator function of λ, let us consider (following A § 5 and 6) an expression for $\mathbf{R}$ of the type

† One can define instead of the resolvent $\mathbf{R}$, the operator $\mathbf{V} = 1 + \lambda \mathbf{R}$. Equations (1.2) and (1.3) are replaced by the equations

$$\mathbf{V} = 1 + \lambda \mathbf{K}\mathbf{V}, \qquad X = \mathbf{V} X_0.$$

$$\mathbf{R}(\lambda) = \frac{\sum_0^N \frac{(-\lambda)^n}{n!} \mathscr{D}_{n+1}}{\sum_0^N \frac{(-\lambda)^n}{n!} D_n} + \mathbf{R}'(\lambda). \tag{1.4}$$

In other words we are trying to approximate $\mathbf{R}$ by a solution of Fredholm type, with polynomials in λ (instead of series). The introduction of (1.4) into (1.2) shows that the identification of the terms of both sides may be made such that one obtains the following set of formulae†:

$$\begin{aligned} \mathscr{D}_1 &= \mathbf{K}, \\ &\cdots\cdots\cdots\cdots \\ \mathscr{D}_p &= D_{p-1}\mathbf{K} - (p-1)\mathbf{K}\mathscr{D}_{p-1}, \qquad (1.5a) \\ &\cdots\cdots\cdots\cdots \\ \mathscr{D}_{N+1} &= D_N\mathbf{K} - N\mathbf{K}\mathscr{D}_N, \end{aligned}$$

$$\mathbf{R}' = -\frac{(-\lambda)^{N+1}}{N!} \frac{\mathbf{K}\mathscr{D}_{N+1}}{\sum_0^N \frac{(-\lambda)^n}{n!} D_n} \alpha + \lambda \mathbf{K}\mathbf{R}'. \tag{1.5b}$$

If we chose the D_n as c-numbers (the $\mathscr{D}_n$ are then operators), the set of formulae (1.5a) enables us to calculate $\mathscr{D}_1 \ldots \mathscr{D}_{N+1}$, for any arbitrary set of D_n, by means of the following formula:

$$\frac{\mathscr{D}_p}{(p-1)!} = (-1)^p \sum_{\varrho=0}^{p-1} (-1)^{\varrho+1} \frac{D_\varrho}{\varrho!} \mathbf{K}^{p-\varrho}. \tag{1.6}$$

On the other hand the equation (1.5b) is then the equation for the remainder $\mathbf{R}'$ which is the difference between the true solution and the approximate one.

Since λ is not an eigenvalue, this difference $\mathbf{R}'$ will be negligible if the c-numbers D_n are chosen such that the inhomogeneous term in (1.5b), i.e.

$$-\frac{(-\lambda)^{N+1}}{N!} \frac{\mathbf{K}\mathscr{D}_{N+1}}{\sum_0^N \frac{(-\lambda)^n}{n!} D_n} \equiv \frac{\lambda^{N+1} \sum_0^N (-1)^\varrho \frac{D_\varrho}{\varrho!} \mathbf{K}^{N+2-\varrho}}{\sum_0^N \frac{(-\lambda)^n}{n!} D_n} \tag{1.7}$$

(as computed from (1.6)), is "small" enough.

Including (1.7) (i.e. zeroth approximation of $\mathbf{R}'$) in the solution, we have the following expression for U:

† In the conventional Fredholm method, all the D_n are independent of λ; it is then sufficient to identify on both sides the coefficients of the same powers of λ.

$$U = U_0 + \lambda \frac{\sum_0^N \frac{(-\lambda)^n}{n!} \mathscr{D}_{n-1} + \lambda^{N+1} \sum_0^N (-1)^\varrho \frac{D_\varrho}{\varrho!} \mathbf{K}^{N+2-\varrho}}{\sum_0^N \frac{(-\lambda)^n}{n!} D_n} U_0. \quad (1.8)$$

We shall consider the question of the size of (1.7) in the next section. Let us conclude these considerations with some general comments. First of all, the connection between the present formalism and Fredholm's method (as introduced in A) may be given as follows. One can show (A, eq. (6.9)) that if the traces of the powers of the kernel **K** do exist, then, with the following choice of the c-numbers D_n:

$$D_n = \operatorname{tr} \mathscr{D}_n = \sum_\alpha \langle \alpha | \mathscr{D}_n | \alpha \rangle, \quad (1.9)$$

$|\alpha\rangle$ is a complete orthonormal system) all the eigenvalues are given by the roots of the denominator.

Furthermore, one may prove as in A that if certain classes of traces are infinite or indeterminate, all these infinities can be discarded and replaced by zeros in the expression for U.

We may also point out that one can extend the Fredholm form of solution by proving that if the series of successive approximations is summable in Nørlund's sense, its sum is still a solution of (1.1) (A § 10).

Finally, it is useful to consider the expression for **R** in (1.4) as given by a certain weighting of the series of successive approximations. One may prove (A, eq. (10.9)) that if we define the rational functions

$$\bar{\omega}_p^{(N)}(\lambda) = \frac{\sum_0^p \frac{(-\lambda)^\nu}{\nu!} D_\nu}{\sum_0^N \frac{(-\lambda)^\nu}{\nu!} D_\nu} \quad (1.10)$$

the expression for **R** takes the following form

$$\mathbf{R}(\lambda) = \sum_{\varrho=0}^N \bar{\omega}_{N-\varrho}^{(N)} \mathbf{K}^{\varrho+1} + \mathbf{R}'. \quad (1.11)$$

2. The Determination of the c-Numbers $\mathbf{D_n}$

Let us first of all notice that the perturbative solution corresponds to the special choice

$$D_0 = 1;\ D_1 = D_2 = \ldots D_N = 0. \tag{2.1}$$

The resolvent $\mathbf{R}$ then takes the form

$$\mathbf{R} = \sum_0^N \lambda^n \mathbf{K}^{n+1} + \mathbf{R}', \tag{2.2}$$

with the following equation for $\mathbf{R}'$:

$$\mathbf{R}' = \lambda^{N+1} \mathbf{K}^{N+2} + \lambda \mathbf{K}\mathbf{R}'. \tag{2.3}$$

The accuracy of the perturbative method is therefore tied up with the size of the term $\lambda^{N+1}\mathbf{K}^{N+2}$ and one may always compare the remainder given by (1.7) to this term in order to find out if there is any advantage in considering methods more complicated than the perturbative one.

The next problem is how to define the "size" of this remainder (1.7). In order to be more specific we shall consider a scattering problem of field theory: the initial state $|\mathrm{I}\rangle$ is, for instance, an n-particle state, and we have to calculate the projection of the state vector $U|\mathrm{I}\rangle$ on the final state $\langle \mathrm{II}|$ defined in the same way as the initial one.

We have therefore to choose the D_n such that the matrix elements $\langle \mathrm{II}|\mathbf{R}'|\mathrm{I}\rangle$ are small for any final state $\langle \mathrm{II}|$ and this condition will be fulfilled if the length of the vector $\mathbf{R}'|\mathrm{I}\rangle$ is small too.

As stated in the former section, *if λ is not an eigenvalue* the length of the vector $\mathbf{R}|\mathrm{I}\rangle$ is tied up with the length of the operator (1.7) applied to the initial state vector† $|\mathrm{I}\rangle$, i.e.

$$\left\| \frac{\lambda^{N+1} \sum_0^N (-1)^\varrho \frac{D_\varrho}{\varrho!} \mathbf{K}^{N+2-\varrho}}{\sum_0^N \frac{(-\lambda)^n}{n!} D_n} |\mathrm{I}\rangle \right\|^2 = \lambda^{2N+2} \frac{\| \sum_0^N x_\varrho X_{N+2-\varrho} \|^2}{|\sum_0^N \lambda^n x_n|^2} \tag{2.4}$$

with

$$x_\varrho = (-1)^\varrho \frac{D_\varrho}{\varrho!}, \qquad X_p = \mathbf{K}^p |\mathrm{I}\rangle. \tag{2.5}$$

The expression (2.4) is therefore that whose minimum gives the

† This point may be rigorously proved if the kernel $\mathbf{K}$ and therefore the resolvent is a bound operator. Let us call $\mathbf{A}$ the operator (1.7); eq. (1.5) takes the form

$$\mathbf{R}' = \mathbf{A} + \lambda \mathbf{K}\mathbf{R}',$$

and

$$|\langle \mathrm{II}|\mathbf{R}'|\mathrm{I}\rangle| = |\langle \mathrm{II}|(1 + \lambda\mathbf{R})\,\mathbf{A}|\mathrm{I}\rangle| \leq \|(1 + \lambda\mathbf{R})\,\mathbf{A}|\mathrm{I}\rangle\| \leq C\|\mathbf{A}|\mathrm{I}\rangle\|$$

since for every bounded operator Ω, one has $\|\Omega X\| \leq C\|X\|$ with C independent of X.

best approximation. Taking derivatives and using the conditions $\Sigma x_n \lambda^n \neq 0$ one obtains without any difficulty the following set of equations:

$$\sum_{\varrho,\varrho'=0}^{N} x_\varrho^* x_{\varrho'} \{\lambda^\varrho \langle X_{N+2-\alpha}|X_{N+2-\varrho'}\rangle - \lambda^\alpha \langle X_{N+2-\varrho}|X_{N+2-\varrho'}\rangle\} = 0 \quad (2.6)$$

$$\alpha = 1 \ldots N$$

and their complex conjugates, i.e. $2N$ equations. They determine $x_1 \ldots x_N$ which are now functions of λ and of the parameters defining the initial state $|\mathrm{I}\rangle$.

Taking, for instance, $N = 1$ (hence $\alpha = 1$ and $\varrho, \varrho' = 0,1$), one easily finds

$$x_1 = -\frac{\langle X_2 - \lambda X_3|X_3\rangle}{\langle X_2 - \lambda X_3|X_2\rangle}. \quad (2.7)$$

In certain cases it will be possible to find a system of D_n for which the term $\mathbf{K}\mathscr{D}_{n+1}$ vanishes: then the solution of (1.1) is given exactly by the ratio of two polynomials†. This is always the case when $\mathbf{K}$ is defined in a space of a finite number of dimensions, as one may prove by a straightforward extension of the Hamilton-Cayley theorem. In the following sections, we shall give several applications of this formalism [2]).

3. S Matrix Theory

Considering a given quantal system in the interaction representation, the S matrix is defined as usual by

$$S(t) = 1 - ig\int_{-\infty}^{+\infty} \theta_+ (t - t')\, H(t')\, S(t')dt', \quad (3.1a)$$

$$S = \lim_{t=\infty} S(t). \quad (3.1b)$$

The representation of S by a power series in g may be written as follows:

$$S = \Sigma g^n S_n, \quad (3.2a)$$

$$S_n(t) = (-i)^n \int_{-\infty}^{+\infty} dt_n \ldots \int_{-\infty}^{+\infty} dt_1 \theta_+ (t - t_n) H(t_n) \ldots \theta_+ (t_2 - t_1) H(t_1), \quad (3.2b)$$

$$S_n = \lim_{t=\infty} S_n(t) = (-i)^n \int_{-\infty}^{+\infty} dt_n \ldots \int_{-\infty}^{+\infty} dt_1 H(t_n) \ldots \theta_+ (t_2 - t_1) H(t_1). \quad (3.2c)$$

† An example is given in section 6 of this paper.

As is well known, S_n is represented by the contribution of all Feynman graphs of order n.

We may, by using the formula (1.6) express $\mathscr{D}_{n+1}(t) \cdot 1$ in terms of the $S_n(t)$. Taking its limit for $t=\infty$, we have to replace $S_m(t)$ by S_m and

$$\lim_{t=\infty} \frac{\mathscr{D}_{n+1}(t) \cdot 1}{n!} = (-1)^{n+1} \sum_0^n (-1)^{\varrho+1} \frac{D_\varrho}{\varrho!} S_{n+1-\varrho} = (-1)^n \sum_0^n x_\varrho S_{n+1-\varrho}. \tag{3.3}$$

Finally the formula (1.8) gives the following expression for the operator S:

$$S = 1 + \frac{\sum_0^N g^{n+1} \sum_0^n x_\varrho S_{n+1-\varrho} + g^{N+2} \sum_0^N x_\varrho S_{N+2-\varrho}}{\sum_0^N g^n x_n}. \tag{3.4}$$

In order to determine the numbers x_n, we have to minimize the term

$$\frac{\| \sum_0^N x_\varrho S_{N+2-\varrho} |\mathrm{I}\rangle \|^2}{|\sum_0^N g^n x_n|^2} \tag{3.5}$$

where $|\mathrm{I}\rangle$ represents the initial state.

It should be noted now that the behaviour of $\langle \mathrm{II}|S|\mathrm{I}\rangle$ for a scattering process is completely different for energies above and below the threshold of production of new particles. In the first case matrix elements like $\langle \mathrm{I}|S_3 S_3|\mathrm{I}\rangle$ are different from zero, while in the second case they vanish. In this last case the results become rather simple.

Let us consider for simplicity the approximation of S up to g^4:

$$S = 1 + \frac{gS_1 + g^2(S_2 + x_1 S_1) + g^3(S_3 + x_1 S_2 + x_2 S_1) + g^4(S_4 + x_1 S_3 + x_2 S_2 + x_3 S_1)}{1 + g x_1 + g^2 x_2 + g^3 x_1}. \tag{3.6}$$

This expression takes for energies below the threshold the following form:

$$\langle \mathrm{II}|S|\mathrm{I}\rangle = \frac{\langle \mathrm{II}|g^2 S_2 + g^3 x_1 S_2 + g^4(S_4 + x_2 S_2)|\mathrm{I}\rangle}{1 + g x_1 + g^2 x_2 + g^3 x_1}. \tag{3.7}$$

and the expression which has to be minimized is

$$\frac{\|(S_4 + x_2 S_2)|\mathrm{I}\rangle\|^2}{|1 + gx_1 + g^2 x_2 + g^3 x_3|^2}. \tag{3.8}$$

It is appropriate to take $x_1 = x_3 = 0$ and the result is easily obtained by using formulae (2.6) or by direct calculation:

$$x_2 = -\frac{\langle \mathrm{I}|S_4(S_2 - g^2 S_4)|\mathrm{I}\rangle}{\langle \mathrm{I}|S_2(S_2 - g^2 S_4)|\mathrm{I}\rangle}. \tag{3.9}$$

Since the S matrix has matrix elements only between states with the same energy, the product of operators in (3.9) corresponds just to integration over angles. By using a mean value theorem for these integrals, one sees that an approximation to (3.9) is

$$x_2 = -\frac{\langle \mathrm{I}|S_4|\mathrm{I}\rangle}{\langle \mathrm{I}|S_2|\mathrm{I}\rangle}, \tag{3.10}$$

a result already obtained by Cini and Fubini [2,3]).

We may conclude this section by saying a few words about the renormalization and the accuracy of this approximation. Renormalization has to be done, by using Dyson prescriptions for instance, in the formulae (3.2c). It is then clear that if the renormalized expansion of the S matrix has a radius of convergence which is different from zero (cf. for discussion A, § 12), the method we gave will certainly work. In case of asymptotic convergence or in the case of divergence the problem has to be examined carefully, but in all cases the accuracy of the method is measured by the length of the following vector in Hilbert space:

$$\frac{(S_4 + x_2 S_2)|\mathrm{I}\rangle}{1 + g^2 x_2} \tag{3.11}$$

where x_2 has the value (3.9).

4. The Reactance Operator

It is well known that by introducing the operators

$$\overline{U}(t) = 1 - \frac{i}{2} g \int_{-\infty}^{+\infty} \varepsilon(t - t') H(t') \overline{U}(t') dt', \tag{4.1a}$$

$$K = g \int_{-\infty}^{+\infty} H(t) \overline{U}(t) dt, \tag{4.1b}$$

one can show that the operators S and $T = S - 1$ satisfy the equations

$$S = \frac{1 - \frac{i}{2}K}{1 + \frac{i}{2}K}, \tag{4.2a}$$

$$T = -iK - \frac{i}{2}KT. \tag{4.2b}$$

Since the only non-vanishing elements of K are on the energy shell, (4.2) can be converted into an ordinary integral equation with integration over angles, i.e. Heitler's integral equation, by considering a special coordinate system. One easily sees, by carrying out the calculation step by step, that the approximations of $\bar{U}$ and K can be expressed by the antihermitian part of the S_n. The shortest way for this result starts from (4.2a): since K is hermitian, taking the hermitian conjugate of (4.2a) and subtracting from (4.2a), one finds

$$K = -S^{(ah)} + \tfrac{1}{4}S^{(ah)}K^2 \tag{4.3}$$

where $S^{(ah)}$ is the antihermitian part of S. Let us define

$$K = \sum_0 g^{n+1}K_{n+1}, \tag{4.4a}$$

$$K_{n+1} = \int_{-\infty}^{+\infty} H(t)\,\bar{U}_n(t)\,dt. \tag{4.4b}$$

The introduction of (4.4) into (4.3) leads to the general recurrence formulae

$$\begin{aligned} &K_1 = -S_1^{(ah)},\ K_2 = -S_2^{(ah)}, \ldots \\ &K_{l+1} = -S_{l+1}^{(ah)} + \tfrac{1}{4}\sum_{\lambda+\mu+\nu=l+1} S_\lambda^{(ah)}K_{\mu+1}K_{\nu+1}. \end{aligned} \tag{4.5}$$

Now expressing $\bar{U}(t)$ by means of the formula (1.8) and going back to the definition of K we have

$$K = g\int_{-\infty}^{+\infty} H(t)\,dt + \frac{\sum_0^N g^{n+2}\sum_0^n x_\varrho K_{n+2-\varrho}}{\sum_0^N g^n x_n} \tag{4.6}$$

and the K_n are given by (4.5). The choice of the c-numbers x_n is to be made as in the former section, and from the integral equation (4.2) we shall be able to deduce an expression for the S operator which is unitary in all its approximations.

5. Adiabatic Nuclear Potentials

Following patterns slightly different from those used by Cini and Fubini [3]), we shall now deduce the adiabatic potential acting between two nucleons in a bound state and apply the previous formalism when the coupling constant g is large. It is well known that in the interaction representation we have

$$S(t) = \bar{U}(t)\frac{1+S}{2}, \tag{5.1}$$

where $\bar{U}(t)$ has been defined by (4.1a). Applying the former expression to $\Psi(-\infty)$, we get for the state vector $\Psi(t)$ an inhomogeneous integral equation which reduces for a system in a bound state ($\Psi(+\infty) = \Psi(-\infty) = 0$) to the homogeneous one

$$\Psi(t) = -\frac{i}{2}g\int_{-\infty}^{+\infty}\varepsilon(t-t')\,H(t')\,\Psi(t')\,dt'. \tag{5.2}$$

Let us now define a projection operator η projecting on the subspace of the state vectors corresponding to states with more than two nucleons. Therefore a state with two nucleons is described by

$$\Phi(t) = (1-\eta)\,\Psi(t). \tag{5.3}$$

We may introduce (5.3) in (5.2), the elimination of $\Psi(t)$ leads to the equation †

$$\Phi(t) = -\frac{i}{2}g\int_{-\infty}^{+\infty}(1-\eta)\,\varepsilon(t-t')\,H(t')\,V(t',t'')\,\Phi(t'')\,dt''\,dt', \tag{5.4a}$$

$$V(t,t') = \delta(t-t') - \frac{i}{2}g\int_{-\infty}^{+\infty}\eta\varepsilon(t-t'')\,H(t'')\,V(t'',t')\,dt''. \tag{5.4b}$$

If we now apply the differential operator $i\partial/\partial t$ to (5.4a) and

† Let us consider eq. (1.1b). If ξ is a given projection operator, a given vector X may be decomposed into

$$X = X' + X'' = \xi X + (1-\xi)X.$$

The introduction of this expression in (1,1b) permits the elimination of X'' and gives the following equation for X':

(a) $$X' = (\xi + (1-\xi)\mathbf{K}\mathbf{V}\xi)X_0 + \xi\mathbf{K}\mathbf{V}X'$$

and

(b) $$\mathbf{V} = \mathbf{1} + (1-\xi)\mathbf{K}\mathbf{V}.$$

Eq. (a) gives (5.4a) (without the inhomogenous term), eq. (b) gives (5.4b), if we take $\eta = 1 - \xi$.

neglect all terms in $\partial\eta/\partial t$, we obtain the following equation for the state vector $\Phi(t)$:

$$i\frac{\partial\Phi}{\partial t} = g\int_{-\infty}^{+\infty}(1-\eta)\,H(t)\,V(t,t')\,\Phi(t')\,dt'. \tag{5.5}$$

In order to define a nuclear potential, let us introduce in (5.5) the stationary state vector

$$\Phi(t) = e^{i(H_0-E)t}\Phi \tag{5.6}$$

where Φ is a vector in Hilbert space independent of t. The following eigenvalue equation results:

$$(E-H_0)\Phi = g\int_{-\infty}^{+\infty} e^{-i(H_0-E)t}(1-\eta)\,H(t)\,V(t,t')\,e^{i(H_0-E)t'}dt'\,.\,\Phi. \tag{5.7}$$

Therefore, if we consider the time-independent hermitian operator†

$$W(E) = g\int_{-\infty}^{+\infty} e^{-i(H_0-E)t}(1-\eta)\,H(t)\,V(t,t')\,e^{i(H_0-E)t'}dt'. \tag{5.8}$$

its matrix elements in the configuration space $\mathbf{r}$, when nucleon recoil is neglected, represent the adiabatic potential. The equation for $V(t,t')$ may be treated in a similar way as in § 3 (eq. (3.1a)) and the solution written down as in (3.4).

6. Application to Heitler's Integral Equation

As an application of this formalism to an ordinary integral equation, we shall study the scattering of charged mesons by nucleons using Heitler's integral equation. It is well known that if we consider charged scalar mesons with pseudovector coupling, this equation can be solved exactly for the elastic scattering of π^- by nucleons. It is interesting to verify this result by using the previous formalism.

Let us take this integral equation in a form which has been used by several authors††:

$$T(\mathbf{p},\mathbf{p}_0)=G(\mathbf{p},\mathbf{p}_0)+\frac{i|\mathbf{p}|}{32\pi^2(E+\varepsilon)}\int G(\mathbf{p},\mathbf{p}')(-i\gamma p'+M)T(\mathbf{p}',\mathbf{p}_0)d\Omega'. \tag{6.1}$$

† It is easy to prove the time independence of $W(E)$ by going to the Schrödinger picture, denoting by the suffix s the operators in this representation, one finds

$$W(E) = g(1-\eta_s)H_sV_s(E)$$

where $V_s(E)$ is the Fourier transform of $V_s(t)$.

†† One obtains (6.1) by replacing in (4.2b) T by $2i\pi T$ and K by $-2\pi G$.

E and ε are the energies of the nucleon and the meson, $\mathbf{p}$ and $\mathbf{p}_0$ the common value of their momenta in the final and in the initial state relative to the c.m. system and the linear operator $\mathbf{K}$ is here an integral operator with the kernel

$$\mathbf{K}f \equiv \int G(\mathbf{p}, \mathbf{p}')\,(-i\gamma p' + M) f(\mathbf{p}')\, d\Omega'.$$

For the scattering of negative mesons, $G(\mathbf{p}, \mathbf{p}')$ is independent of the angles and depends only on the energies E and ε, i.e. $G(\mathbf{p}, \mathbf{p}') = G(\mathbf{p}, \mathbf{p}_0)$. The first approximation of T (corresponding to $\mathbf{K}U$) is

$$\mathbf{K}U \to 4\pi i G(\mathbf{p}, \mathbf{p}_0)\,(\gamma_4 E + M)\, G(\mathbf{p}, \mathbf{p}_0), \tag{6.2}$$

while the second (corresponding to $\mathbf{K}^2U$) is

$$\mathbf{K}^2U \to (4\pi i)^2 G(\mathbf{p}, \mathbf{p}_0)\,(\gamma_4 E + M)\, G(\mathbf{p}, \mathbf{p}_0)\,(\gamma_4 E + M)\, G(\mathbf{p}, \mathbf{p}_0). \tag{6.3}$$

Introducing, by means of eq. (1.5a), the function $\mathscr{D}_2 G(\mathbf{p}, \mathbf{p}_0)$ we have

$$\mathscr{D}_2 G(\mathbf{p}, \mathbf{p}_0) = 4\pi i\, G(\mathbf{p}, \mathbf{p}_0)\,(\gamma_4 E + M)\, G(\mathbf{p}, \mathbf{p}_0)\,\{D_1 - 4\pi i\,(\gamma_4 E + M)\, G(\mathbf{p}, \mathbf{p}_0)\}.$$

Therefore $\mathscr{D}_2 G$ can be taken equal to zero if

$$D_1 = 4\pi i\,(\gamma_2 E + M)\, G(\mathbf{p}, \mathbf{p}_0). \tag{6.5}$$

Choosing $D_2 = D_3 = \ldots\ldots\ldots = 0$ we have $\mathscr{D}_2 G, \mathscr{D}_3 G, \ldots$ equal to zero. The exact solution is then

$$T(\mathbf{p}, \mathbf{p}_0) = \frac{G(\mathbf{p}, \mathbf{p}_0)}{1 - \dfrac{i|\mathbf{p}|}{8\pi(E+\varepsilon)}\,(\gamma_4 E + M)\, G(\mathbf{p}, \mathbf{p}_0)}. \tag{6.6}$$

If we now consider the scattering of π^+ by nucleons, it is well known that there is no exact solution because $G(\mathbf{p}, \mathbf{p}_0)$ depends in fact on the angles made by $\mathbf{p}_0$ with the coordinate axes. An approximate solution which is stopped at D_1 may be written as follows:

$$T(\mathbf{p}, \mathbf{p}_0) = \frac{1}{1 - \dfrac{|\mathbf{p}|}{32\pi^2(E+\varepsilon)}} \Bigg\{ G(\mathbf{p}, \mathbf{p}_0) + \frac{\mathbf{p}}{32\pi^2(E+\varepsilon)} \left(i \int G(\mathbf{p}, \mathbf{p}')\,(i\gamma p' + M) G(\mathbf{p}', \mathbf{p}_0)\, d\Omega' - D_1 G(\mathbf{p}, \mathbf{p}_0) \right) \Bigg\}. \tag{6.7}$$

In terms of the operator $\mathbf{K}$ and of the scalar product in the functional space of the angles, we have from (2.7)

$$D_1 = -\frac{\langle K^2G - gK^3G, K^3G\rangle}{\langle K^2G - gK^3G, K^2G\rangle}, \tag{6.8}$$

We see that a rather rough approximation of D_1 from (6.8), is given by

$$D_1 = \frac{i}{4\pi}\int G(\mathbf{p}, \mathbf{p}')(i\gamma p' + M)\, d\Omega\, d\Omega'. \tag{6.9}$$

All the integrations are elementary and can be worked out by using some previous calculations made by S. N. Biswas[4]). The results obtained by using this method do not differ essentially from those given by the former author.

We shall not go deeper into this problem, since we consider it to be a rather simple application of our formalism.

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COMMUTATION RELATIONS AND RELATIVISTIC WAVE EQUATIONS

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Abstract: Covariant commutation relations for particles of any spin are given in the cases where the particles have only one mass state or have a mass spectrum.

1. General Theory

In a recent paper [1]), S. N. Gupta studied the quantization of particles of spin 3/2, by starting from equations whose form is different from the Rarita-Schwinger [2]) equations. We wish to give a simple general formulation which leads to Gupta's equations in the special case of spin 3/2.

We assume here that the equation for the free field $Q(x)$ is

$$\Lambda(\partial)Q(x)\equiv(\varrho_\mu\partial_\mu+\beta\varkappa)Q(x)=0, \qquad (1)$$

where $Q(x)$ is a one column matrix of N elements $Q_A(x)$: $(A=1\ldots N)$; ϱ_μ $(\mu=1,2,3,4)$ and β are $N\times N$ matrices.

In order to determine the commutation relations for the field $Q(x)$, we are going to solve the following problem: "How do the commutation relations change, when one changes the field variables?"

The equation (1) may be derived from the following action:

$$\mathscr{A}=\int Q^*_A(x)\eta_{AA'}\Lambda_{A'B}(\partial)Q_B(x)dx\equiv\int Q^*(x)\eta\Lambda(-\overleftarrow{\partial})Q(x)dx, \qquad (2)$$

where the second expression for $\mathscr{A}$ has been obtained by integration by parts and $\overleftarrow{\partial}$ acts on the left. The matrix η is a non singular matrix which has been chosen such that the equations deduced

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from (2) by independent variations of Q and Q^* are compatible. Therefore we have

$$\eta\Lambda(\partial)=(\eta\Lambda(-\partial))^{\dagger}, \tag{3}$$

where the symbol $\dagger$ means the hermitian conjugate.

Let us change the wave functions $Q(x)$ as follows:

$$Q_A(x)=\mathscr{D}_{AA'}(\partial)q_{A'}(x); \tag{4}$$

$\mathscr{D}(\partial)$ is a matricial differential operator . Equation (1) then takes the following form:

$$\begin{aligned}\mathscr{A}&=\int(\mathscr{D}^*_{A''A'}(\partial)q^*_{A'}(x))\eta_{A''A}\Lambda_{AB}(\partial)\mathscr{D}_{BB'}(\partial)q_{B'}(x)dx\\&=\int q^*_{A'}(x)\mathscr{D}^*_{A''A'}(-\partial)\eta_{A''A}\Lambda_{AB}(\partial)\mathscr{D}_{BB'}(\partial)q_{B'}(x)dx.\end{aligned} \tag{5}$$

The bracket in the first line of eq. (5) means that $\mathscr{D}^*(\partial)$ acts only on $q^*(x)$, and the second expression for $\mathscr{A}$ has been deduced by integration by parts.

Introducing the hermitian conjugate of the matrix $\mathscr{D}$, the formula (5) takes the form

$$\mathscr{A}=\int q^*(x)\mathscr{D}^{\dagger}(-\partial)\eta\Lambda(\partial)\mathscr{D}(\partial)q(x)dx \tag{6}$$

and it follows that $q(x)$ satisfies

$$\mathscr{D}^{\dagger}(-\partial)\eta\Lambda(\partial)\mathscr{D}(\partial)q(x)=0. \tag{7}$$

We now suppose that the commutation (or anticommutation) relations for the field $q(x)$ are known:

$$c_{AB}(x-x')=[q_A(x),q^*_B(x')]_{\pm}, \tag{8}$$

and we try to deduce the commutation relations for the field $Q(x)$:

$$C_{AB}(x-x')=[Q_A(x),\overline{Q}_B(x')], \tag{9}$$

where

$$\overline{Q}_A(x)=Q^*_{A'}(x)\eta_{A'A}.$$

Using (4) we obtain the following formula:

$$C_{AB}(x-x')=\mathscr{D}_{AA'}(\partial)\mathscr{D}^*_{B'B''}(\partial')c_{A'B''}(x-x')\eta_{B'B}, \tag{10}$$

where ∂' acts on x'. Equation (10) can also be written in matrix form

$$C(x)=\mathscr{D}(\partial)c(x)\mathscr{D}^{\dagger}(-\partial)\eta. \tag{11}$$

Thus, the question which was asked before may be considered as answered, provided that the fields $Q(x)$ and $q(x)$ are compatible

† We can define a differential operator $F(\partial)$ by its Fourier transform

$$F(\partial)f(x)=\int\exp(ik_\mu x_\mu)\,F(ik_\mu)f(ik_\mu)dk,$$

with $f(x)$ an arbitrary derivable function of x

(cf. §3 for supplementary conditions introduced by the consideration of the spin of these two fields).

These formulae can be simplified to a great extent if we take a field $q(x)$ such that equation (7) reduces to the equation

$$\mathscr{D}^{\dagger}(-\partial)\eta\Lambda(\partial)\mathscr{D}(\partial)q(x)\equiv f(\partial)q(x),$$

where $f(\partial)$ is a scalar differential operator ($\Box-\varkappa^2$ for a one mass-level field, $\Pi'_{i=1}(\Box-\varkappa_i^2)$ for a multimass field), and that $c(x)$ is a diagonal matrix.

We may then look for a differential operator $D(\partial)$ such that

$$\Lambda(\partial)D(\partial)=f(\partial)I. \tag{12}$$

It can easily be seen by using the scalar character of the right hand side that

$$D(\partial)=\mathscr{D}(\partial)\mathscr{D}^{\dagger}(-\partial)\eta. \tag{13}$$

Therefore, if one goes back to the expression (11) for $C(x)$, one has

$$C(x)=D(\partial)c(x). \tag{14}$$

Let us now show that the matrix β in (1) is a non singular matrix, if we assume the existence of the differential operator $D(\partial)$. Indeed, we may write down an expression for this operator as follows:

$$D(\partial)=\alpha+\alpha_{\sigma}\partial_{\sigma}+\alpha_{\sigma_1\sigma_2}\partial_{\sigma_1}\partial_{\sigma_2}+\ldots, \tag{15}$$

where the α, α_{σ_1}, $\alpha_{\sigma_1\sigma_2}$ are $N\times N$ matrices and the suffices σ_j run from 1 to 4. We introduce this expression into equation (12) and identify terms of the same derivative order on both sides. If we suppose that the expression for $f(\partial)$ contains a term M independent of ∂, we finally obtain

$$\beta\alpha=MI. \tag{16}$$

The matrix β is therefore non singular. Defining the matrices ϱ_{μ} as follows:

$$\beta_{\mu}\equiv\beta^{-1}\varrho_{\mu}, \tag{17}$$

equation (1) can be written

$$(\beta_{\mu}\partial_{\mu}+\varkappa)Q(x)=0. \tag{18}$$

2. One-mass Equation

In all problems connected with particles with only one mass, we have to require that one possible form $f(\partial)$ is $\Box-\varkappa^2$, and $q(x)$ then is a scalar field (with several components). Thus the canonical

formalism leads to the well-known commutation relations for the field variable $q_A(x)$:

$$[q_A(x), q_B^*(x')]_\pm = i\Delta(x-x')\delta_{AB}, \tag{19}$$

and by means of the transformation (14)

$$[Q_A(x), \overline{Q}_B(x')]_\pm = iD_{AB}(\partial)\Delta(x-x'). \tag{20}$$

The commutation relations are therefore completely determined by calculating $D(\partial)$ from (13).

Moreover, we may notice that only $2S+1$ components $Q_A(k)$ (Fourier amplitudes of $Q_A(x)$) are not zero when $k_\mu^2+\varkappa^2=0$, since a field of spin S is described by $2S+1$ independent components. We conclude therefore from (20) that the rank of the matrix $D(\partial)\Delta(x)$ is $2S+1$.

Let us write now the operator $D(\partial)$ as in equation (15):

$$D(\partial)=\Sigma_{j=0}\alpha_{\sigma_1\ldots\sigma_j}\partial_{\sigma_1}\ldots\partial_{\sigma_j}. \tag{21}$$

We may suppose, without any loss of generality, that the $\alpha_{\sigma_1\ldots\sigma_j}$ are symmetrical with respect to the exchange of their indices. With $\Lambda\equiv\beta_\mu\partial_\mu+\varkappa$, the relation (12) leads to recurrence formulae which determine the coefficients α_σ:

$$\begin{aligned} &\text{a) } \alpha_0=-\varkappa I, \quad \text{b) } \alpha_0\beta_\mu+\varkappa\alpha_\mu=0, \quad \text{c) } \mathbf{S}(\beta_\mu\alpha_\nu+\varkappa\alpha_{\mu\nu})=2\delta_{\mu\nu}, \\ &\text{d) } \mathbf{S}(\beta_{\sigma_1}\alpha_{\sigma_2\ldots\sigma_{p-1}}+\varkappa\alpha_{\sigma_1\ldots\sigma_p})=0, \qquad p>2; \end{aligned} \tag{22}$$

S denotes a summation over all possible permutations with respect to the suffices. The solutions of the equations (22) can be easily worked out:

$$\alpha_\mu=\beta_\mu, \tag{23}$$

$$\alpha_{\mu\nu}=\frac{1}{\varkappa}\delta_{\mu\nu}-\frac{1}{2\varkappa}\mathbf{S}\beta_\mu\beta_\nu, \tag{24}$$

. . . . . .

$$\alpha_{\sigma_1\ldots\sigma_p}=\frac{1}{\varkappa^{p-1}}\frac{1}{p!}\mathbf{S}\beta_{\sigma_1}\ldots\beta_{\sigma_{p-2}}(\delta_{\sigma_{p-1}\sigma_p}-\beta_{\sigma_{p-1}}\beta_{\sigma_p}). \tag{25}$$

Thus, we have

$$\begin{aligned} D(\partial)=&-\varkappa+\beta_\mu\partial_\mu+\frac{1}{\varkappa}(\Box-\beta_\mu\beta_\nu\partial_\mu\partial_\nu)+\ldots \\ &+\frac{1}{\varkappa^{p-1}}\beta_{\sigma_1}\ldots\beta_{\sigma_{p-2}}(\Box-\beta_{\sigma_{p-1}}\beta_{\sigma_p}\partial_{\sigma_{p-1}}\partial_{\sigma_p})\partial_{\sigma_1}\ldots\partial_{\sigma_{p-2}}. \end{aligned} \tag{26}$$

We can now prove, for a field of maximum spin S, that the expansion of (26) must stop at the term whose order is $2S$ in the

derivative operators. At least one of the coefficients $\alpha_{\sigma_1 \ldots \sigma_{2S}}$ is different from zero, and

$$\alpha_{\sigma_1 \ldots \sigma_p}=0 \qquad \text{for } p>2S. \tag{27}$$

The $Q_A (A=1 \ldots N)$ span a space yielding a representation of the 3-dimensional rotation group which can be reduced into irreducible representations D_S, D_{S-1}, ... Therefore, the representation whose basis is given by the direct product $Q_A Q_B$ can be split into D_{2S}, D_{2S-1}, ... (Clebsch-Gordan theorem).

Similarly, since $\Delta(x)$ is a scalar, $\partial_{\mu_1} \ldots \partial_{\mu_p} \Delta(x)$ corresponds to a set of representations D_p, D_{p-1}, ... except when some of the ∂_μ build up dalembertians.

Thus, $\alpha_{\sigma_1 \ldots \sigma_p} \partial_{\sigma_1} \ldots \partial_{\sigma_p}$ $(p>2S)$ can appear only in the form

$$\mathbf{S}\alpha'_{\sigma_1 \ldots \sigma_{2S}} (\Box)^{\frac{1}{2}(p-2S)} \partial_{\sigma_1} \ldots \partial_{\sigma_{2S}} \tag{28}$$

and the recurrence formula (22d) gives

$$\mathbf{S}\alpha'_{\sigma_1 \ldots \sigma_{2S}}=0, \tag{29}$$

which shows that $\alpha_{\sigma_1 \ldots \sigma_p}=0$ for $p>2S$.

We finally deduce from (27) that the β matrices for a field of maximum spin S satisfy the equations

$$\mathbf{S}\beta_{\sigma_1} \ldots \beta_{\sigma_{2S-1}} (\delta_{\sigma_{2S}, \sigma_{2S+1}} - \beta_{\sigma_{2S}} \beta_{\sigma_{2S+1}})=0. \tag{30}$$

Taking $S=1/2$, we obtain the anticommutation relations of Dirac theory. If we take $S=1$, equation (30) admits, as a solution, the β_μ of the Duffin-Kemmer-Petiau theory. For $S=3/2$, one obtains the relation given by S. N. Gupta.

Therefore, we can conclude that the β_μ must satisfy (30) in order to give a quantized field theory of equation (7) and then, their commutation relations can be given by (20) and (26). Harish-Chandra [4]) obtained an equation similar to (30) by means of purely algebraical methods. His equation is of highest order n in β, where n is just the order of the minimal equation of β_μ. Our discussion gives a physical meaning to this number n:

$$n=2S+1, \tag{31}$$

by requiring the canonical quantization of fields.

Indeed, the minimal equation derived from (30) is

$$\beta_\mu^{2S-1}(1-\beta_\mu^2)=0. \tag{32}$$

As pointed out by the same author, (30) does not generate a finite

algebra. Some stronger tensor condition compatible with (30) is needed in order to make this algebra finite. An example of this is found in the Duffin-Kemmer-Petiau theory. However, whatever the new condition is, the commutation relations must be given by (20) and (26) with $p=2S$.

3. Multimass Equations

We now extend our results to the case where the wave equation (1) leads to multimass equations †

$$(\Box-\varkappa_1^2)\ldots(\Box-\varkappa_j^2)Q_A(x)=0. \tag{33}$$

As in the former case, there must exist a differential operator $D(\partial)$ defined as follows:

$$D(\partial)\Lambda(\partial)=\Pi_{j=1}^{f}(\Box-\varkappa_j^2). \tag{34}$$

We can prove, by using the same arguments as in section 2, that the commutation relations are

$$[Q_A(x),\overline{Q}_B(x')]_{\pm}=iD_{AB}(\partial)\Delta(x-x'), \tag{35}$$

where $\Delta(x)$ is a scalar function satisfying ††

$$\Pi_{j=1}^{f}(\Box-\varkappa_j^2)\Delta(x)=0, \tag{36}$$

and

$$\Delta(x)=0 \tag{37}$$

for any space like vector x_μ.

Writing down the wave equation in the form (18) and substituting $D(\partial)$ in (34), we have

$$\alpha_0=\frac{(-1)^f}{\varkappa}\Pi_{j=1}^{f}\varkappa_j^2, \tag{38a}$$

$$\alpha_\sigma=\frac{(-1)^{f-1}}{\varkappa^2}\Pi_{j=1}^{f}\varkappa_j^2\beta_\sigma, \tag{b}$$

$$\alpha_{\sigma_1\sigma_2}=\frac{(-1)^f}{2!\varkappa^3}\Pi_{j=1}^{f}\varkappa_j^2\,\mathbf{S}\left(\beta_{\sigma_1}\beta_{\sigma_2}-\sum_{i=1}^{f}\frac{\varkappa^2}{\varkappa_i^2}\delta_{\sigma_1\sigma_2}\right), \tag{c}$$

. . . . .

The discussion given in the former section about the highest derivative order in $D(\partial)$ remains valid: this order must be $2S$.

† As is well-known, the energy operator is not positive definite in such a case.

†† An explicit form of this function has been given for instance by Y. Takahashi and H. Umezawa [5]).

As an example consider the case $f=2S$, S integer; $\varkappa_j=\varkappa/j$. Since $\alpha_{\sigma_1\ldots\sigma_{2S+1}}$ is zero,

$$0=\alpha_{\sigma_1\ldots\sigma_{2S+1}}=a\mathbf{S}(\beta_{\sigma_1}\beta_{\sigma_2}-\delta_{\sigma_1\sigma_2})(\beta_{\sigma_3}\beta_{\sigma_4}-(2)^2\delta_{\sigma_3\sigma_4}) \times\ldots(\beta_{\sigma_{2S-1}}\beta_{\sigma_{2S}}-S^2\delta_{\sigma_{2S-1},\sigma_{2S}})\beta_{2S+1}, \tag{39}$$

where a is a constant (c-number).

This relation leads to the following minimal equation for the β_μ:

$$(\beta_\mu^2-1)(\beta_\mu^2-4)\ldots(\beta_\mu^2-S^2)\beta_\mu=0, \tag{40}$$

which has been given by Bhabha [6]).

In the case of half integer S, if we choose

$$f=S+\tfrac{1}{2}, \tag{41}$$

and

$$\varkappa_j=\frac{2\varkappa}{2j+1}, \tag{42}$$

we have for the minimal equation:

$$(\beta_\mu^2-\tfrac{1}{4})(\beta_\mu^2-\tfrac{9}{4})\ldots(\beta_\mu^2-S^2)=0. \tag{43}$$

This is again a relation given by Bhabha †.

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† The generating operator $S_{\mu\nu}$ of the rotation group has been given by K. K. Gupta [7]) for the special case $S=3/2$. The expression differs from $S_{\mu\nu}=g[\beta_\mu,\beta_\nu]_-$ because this last leads to a multimass theory.

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MESIC AND NUCLEONIC CONTRIBUTIONS TO THE ANOMALOUS MAGNETIC MOMENTS

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Abstract: A rule is given, enabling to classify, without ambiguity, every term as "nucleonic" or "mesic" contribution.

In some recent discussions of the anomalous magnetic moments of the nucleon [1,2]), it has been pointed out that the distinction between "nucleon current contribution" and "meson current contribution" is of great importance. For such considerations, however, one must be able unambiguously to recognize the "class" to which a given term, illustrated by a given diagram, belongs. There seems to be some confusion [1,2]) about this point, and the aim of this short note is to provide a mathematical criterion determining, for every term, its belonging either to the one or to the other class. The distinction is made according to the following rule:

Consider each term as a function of $-q^2$ (the 4-dimensional square of the energy momentum of the emitted photon), the nucleon being in its free particle state ($p^2=-m^2$; $(p-q)^2=-m^2$).
Then,

(i) those terms for which branch points, in the photon energy scale, occur at $-q^2=(2n)^2\mu^2$ (μ is the π-meson mass, $n=1,2,3\ldots$) are to be classified in the "mesic current contribution";

(ii) all other terms are to be classified in the nucleonic current contribution.

As an example, we choose a term which was classified in the nucleonic contribution in ref. [1]) and [2]), by being arbitrarily considered as reducible to a second order nucleonic term †. Its analytical expression involves a denominator which is, after use of conventional tricks, proportional to

† Denoted by IIe in ref. [1]).

$$\left\{q^2\left[w(1-v)+(\alpha_{33}-\alpha_{13})vw-\frac{1-v(1-\alpha_{32})w^2(1-v(1-\alpha_{32}+\alpha_{21}))}{1-vw(1-\alpha_{22})}\right]\right.$$
$$\left.+m^2\left[vw(z-\alpha_{11})+\frac{\alpha_{21}^2 w^2 v}{1-vw(1-\alpha_{22})}\right]+\mu^2(1-vwz)\right\}, \quad (1)$$

with $\alpha_{11}=z(1-z)$; $\alpha_{22}=xyz(1-xyz)$; $\alpha_{33}=(1-y)z(1-(1-y)z)$;

$$\alpha_{13}=(1-y)z(1-z);\ \alpha_{32}=xyz^2(1-y);\ \alpha_{21}=xyz(1-z), \quad (2)$$

x, y, z, v and w being Feynman parameters, to be integrated from 0 to 1.

The first branch point in the $(-q^2)$ scale occurs at the maximum of the q^2-coefficient for which the coefficient of m^2 vanishes. This maximum is reached for $z=0$ (for which value all the $\alpha_{ik}=0$), $v=0$, $w=\frac{1}{2}$. The q^2-coefficient is then $w(1-w)=\frac{1}{4}$, and (1) becomes

$$\{\tfrac{1}{4}q^2+\mu^2\}, \quad (3)$$

with a zero point at $q^2=-4\mu^2$, denoting the existence of a branch point of the kind $(2n)^2\mu^2$.

Diagrammatically, this classification comprises in the nucleonic contribution every diagram in which the photo emission takes place directly on the continuous nuclear line while any other diagram falls into the mesic contribution †. According to ref. [2]) one can define numerical coefficients B providing the anomalous magnetic moments γ_N and γ_P-1, respectively, for the neutron and the proton:

$$\gamma_N=-\frac{g^2/4\pi}{2\pi}[B_2+B_{1N}];\quad \gamma_P-1=\frac{g^2/4\pi}{2\pi}[B_2-\tfrac{1}{2}B_{1P}]. \quad (4)$$

The numerical results quoted in ref. [2]) are then to be modified as follows

$$\begin{aligned} B_2 &= 0.695-0.009(g2/4\pi);\\ B_{1P} &= 0.947-0.413(g2/4\pi);\\ B_{1N} &= 0.947+0.110(g2/4\pi). \end{aligned} \quad (5)$$

Thanks are due to Professor L. Rosenfeld for enlightening discussions.

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† The same holds for the neutron-electron interaction.

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ON THE ANOMALOUS MAGNETIC MOMENTS

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Abstract: An expression for the nucleonic (electronic) current contribution to the anomalous moment of the nucleon (electron) is derived from the general form of the Green's function. It is shown that these anomalous moments are related to the low energy limit of Compton-like scattering processes. The simplification for practical computations is evident.

The complete nucleonic (electronic) Green's function $S(p)$ satisfies, as is well known, an integro-differential equation[1,2]), whose representation in the Fourier space is †

$$\left[(i\gamma p + m)\varphi_1(p^2) + \frac{p^2 + m^2}{2m}\varphi_2(p^2)\right] S(p) = Z, \qquad (1)$$

where φ_1 and φ_2 are functions of the invariant square length $(p^2)^{-1}$, and also of the masses m and μ of the nucleons (electrons) and π-mesons, and of the interaction constant $g^2/4\pi$ ($e^2/4\pi$); Z is the Green's function normalisation factor.

The nucleonic current contribution to the magnetic moment is contained in the electric current operator $\Gamma_\mu(p', p'')$. One can obtain this whole contribution by limiting the development of $\Gamma_\mu(p', p'')$ in powers of $q_\alpha = (p' - p'')_\alpha$ to the terms linear in q:

$$\Gamma_\mu(p', p'') = \tfrac{1}{2}\,[\Gamma_\mu(p', p') + \Gamma_\mu(p'', p'') + \\ + (p'-p'')_\alpha \partial_{p'_\alpha}\Gamma_\mu(p'=p'', p'') + (p''-p')_\alpha \partial_{p''_\alpha}\Gamma_\mu(p', p''=p') + \ldots]. \qquad (2)$$

The $\Gamma_\mu(p, p)$ with equal arguments can be derived from $S(p)$ since ††

$$e\Gamma_\mu(p, p) = e\,\frac{\partial}{\partial p_\mu} S^{-1}(p). \qquad (3)$$

At this point it must be recalled that the electric current has to be normalised in such a way that, for the free particle state,

† Mass renormalisation already performed.

†† e is the renormalised electric charge.

$$e\Gamma_\mu(p, p)|_{i\gamma p=-m} = e\gamma_\mu, \tag{4}$$

a condition which ensures that the electric charge interacts classically with a slowly variable field. From (3) and (1) we have

$$e\Gamma_\mu(p, p)|_{i\gamma p=-m} = e\gamma_\mu Z^{-1}(\varphi_1(-m^2) + \varphi_2(-m^2)). \tag{5}$$

Hence the normalisation (4) of the electric current fixes Z:

$$Z = \varphi_1(-m^2) + \varphi_2(-m^2). \tag{6}$$

The first two terms $\Gamma_\mu(p', p')$ and $\Gamma_\mu(p'', p'')$ on the right hand side of (2), operating between $\psi(p')$ and $\bar{\psi}(p'')$, give a contribution to the anomalous magnetic moment (a.m.m.), proportional to

$$(e/2m)q_\alpha\sigma_{\mu\alpha}\frac{\varphi_2(-m^2)}{\varphi_1(-m^2) + \varphi_2(-m^2)}; \qquad \sigma_{\mu\alpha} = \frac{i(\gamma_\mu\gamma_\alpha - \gamma_\alpha\gamma_\mu)}{2}. \tag{7}$$

The 3rd and 4th terms of (2), involving the ∂_{p_α} derivatives, are related to the $\Gamma_\mu(p, p)$ of (3) by

$$\partial_{p'_\alpha}\Gamma_\mu(p,' p') = \partial_{p'_\alpha}\Gamma_\mu(p', p'')_{p'=p''} + \partial_{p''_\alpha}\Gamma_\mu(p', p'')_{p''=p'} \tag{8}$$

which is obviously true for the q-independent terms. The latter are just those we want, as only the terms linear in q are needed for the magnetic moment. The non-vanishing terms of the left hand side of (8) independent of q are proportional to

$$(e/2m)\frac{g_{\mu\alpha}\varphi_2(-m^2)}{\varphi_1(-m^2) + \varphi_2(-m^2)} \tag{9}$$

($g_{\mu\alpha}$ is the symmetric tensor), when (8) is applied between $\psi(p')$ and $\bar{\psi}(p'')$, neglecting higher order in q. For each separate term on the right hand side of (8), $\varphi_2(-m^2)$ is decomposed unambiguously into two parts:

$$\varphi_2(-m^2) = 2\beta_1(-m^2) + 2\beta_2(-m^2),$$

in such a way that one has

$$\beta_1(-m^2)\gamma_\alpha\gamma_\mu + \beta_2(-m^2)g_{\mu\alpha} \tag{9a}$$

for the first term of (8),

$$\beta_1(-m^2)\gamma_\mu\gamma_\alpha + \beta_2(-m^2)g_{\mu\alpha} \tag{9b}$$

for the second term.

In (2) the half-difference (9a) − (9b) has to be taken, giving

$$(e/2m)\frac{2\beta_1(-m^2)\sigma_{\alpha\mu}q_\alpha}{\varphi_1(-m^2) + \varphi_2(-m_2)}. \tag{10}$$

Together with (7), (10) provides the total contribution to the a.m.m.

$$(e/2m)q_\alpha\sigma_{\mu\alpha}\frac{2\beta_2(-m^2)}{\varphi_1(-m^2)+2\beta_1(-m^2)+2\beta_2(-m^2)} = q_\alpha\sigma_{\mu\alpha}\lambda_n \qquad (11)$$

where λ_n denotes the nucleonic (electronic) contribution to the a.m.m. in nuclear (Bohr) magneton units.

A closer examination of λ_n shows that it represents the part symmetric in μ and α of the term in the Compton-like effect, in which A_α is absorbed before A_μ is emitted, in its low-energy limit †. From the point of view of an explicit computation in a perturbation expansion, the problem of evaluating magnetic moments is then reduced to that of computing the above effect which is practically similar to a regularised self-energy. Some examples will be given in a subsequent paper.

I would like to thank Professor L. Rosenfeld for many helpful discussions.

† A_μ is the μ-component of the electromagnetic potential.

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RENORMALIZATION WITH PSEUDO-VECTOR COUPLING

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Arnowitt and Deser, in a recent publication [1]) have provided indications that field theories with pseudo-vector coupling may be renormalizable, contrary to what has hitherto often been supposed. Using the functional theory of Edwards and Peierls [2]), they have obtained the renormalized fermion propagator, expressed as an infinite series of integrals in the coordinate representation; and, by a procedure of analytic continuation, shown how to obtain renormalized and presumably finite expressions in the momentum representation also. It is the purpose of this note, firstly, to demonstrate that their renormalization procedure is equivalent to that developed for a wide class of field theories by the author [3]) and, secondly, to point out that finite results can also be obtained with pseudo-vector coupling, by medium coupling methods.

The equation satisfied by the unrenormalized propagator G is

$$\{i\nabla - m + g\gamma_5\nabla(\varphi)\}\, G(x, x') = \delta(x - x'), \tag{1}$$

where m is the unrenormalized fermion mass, $\nabla = \gamma \cdot (\partial/\partial x)$, and γ_5 is chosen so that $\gamma_5^2 = -1$. The meson field variable φ is an operator which can be represented by $\varphi = \varphi_+ + \varphi_-$, where

$$\begin{aligned} \varphi_+ &= \sum_k a_k e^{-ik\cdot x} \\ \varphi_- &= -\sum_k e^{ik\cdot x} \Delta_k (\partial/\partial a_k) \end{aligned} \tag{2}$$

and the a_k are variable c-numbers, of which G is implicitly a function. The substitutions

$$G(x, x') = \exp(ig\gamma_5\varphi)\, H(x, x') \exp\{ig\gamma_5\varphi(x')\} \tag{3}$$

reduces (1) to

$$\{i\nabla - m\exp(2ig\gamma_5\varphi)\}\, H(x, x') = \delta(x - x'). \tag{4}$$

Now the essential result of Arnowitt and Deser's calculations is to make it appear that the infinities of the pseudo-vector coupling are all contained as simple factors in the exponentials $\exp(ig\gamma_5\varphi)$ and $\exp(2ig\gamma_5\varphi)$. It is, in fact, easily shown from (2) that

$$\begin{aligned}\exp(ig\gamma_5\varphi) &= \exp\{\tfrac{1}{2}g^2[\varphi_+, \varphi_-]\}\exp(ig\gamma_5\varphi_+)\exp(ig\gamma_5\varphi_-)\\ &= \exp\{\tfrac{1}{2}ig^2\Delta(0)\}\exp(ig\gamma_5\varphi_+)\exp(ig\gamma_5\varphi_-)\end{aligned} \tag{5}$$

where $i\Delta(0)$ is the divergent sum $\Sigma_k\Delta_k$. The operator $\exp(ig\gamma_5\varphi_-)$ is a "displacement" operator which merely increases the value of each a_k by $ig\gamma_5 e^{ik\cdot x}\Delta_k$ in any expression which follows it. Hence the infinities can be removed by incorporation in the renormalized mass m' and propagator G', defined by

$$\begin{aligned}m' &= m\exp\{2ig^2\Delta(0)\},\\ G' &= Z^{-1}G = \exp\{-ig^2\Delta(0)\}G.\end{aligned} \tag{6}$$

Once this has been seen, the renormalization problem can be stated in a more familiar way. One needs only to eliminate the renormalization constants $\delta m = m' - m$ and Z from the equation (1), written in the form

$$Z\{i\nabla - m' + \delta m + g\gamma_5(\nabla\varphi)\}G'(x, x') = \delta(x - x'), \tag{7}$$

or, in the momentum representation,

$$Z\{\not{p} - m' + \delta m + g\gamma_5\Sigma_k\gamma_k(a_k - \Delta_k\partial/\partial a_k)\}G'(p) = 1 \tag{8}$$

where $\gamma_k = \gamma_5(\gamma\cdot k)E_k$ and E_k is the operator increasing p by k. The author [3]) has shown how to do this, obtaining the typical equations of medium coupling field theory in a renormalized form.

It has been known to the author for some time that the infinities can be eliminated from various medium coupling equations with equal ease whether the coupling is pseudo-scalar or pseudo-vector. To give one simple example of this, consider the equation

$$\begin{aligned}\Gamma(p, k) = Z - \lambda\int &\gamma_5(\mathbf{k} - \mathbf{l})(\mathbf{p} - \mathbf{l} - m)^{-1}\Gamma(p, l)(\mathbf{p} + \mathbf{l} - m)^{-1}\\ &\gamma_5(\mathbf{k} - \mathbf{l})\{(k - l)^2 - \mu^2\}^{-1}d^4l\end{aligned} \tag{9}$$

which is a pseudo-vector equivalent of one discussed by Edwards [4]). For simplicity choose $p = 0$ and set $C(k) = \Gamma(0, k)$. Then scalar solutions $C(k)$ satisfy

$$C(k) = Z + \lambda\int C(l)(l^2 - m^2)^{-1}[1 + \mu^2\{(k - l)^2 - \mu^2\}^{-1}]d^4l. \tag{10}$$

If one takes $Z = -\lambda\int C(l)(l^2 - m^2)^{-1}d^4l$, this is reduced to the

equation treated by Edwards. It now appears that all the infinities of pseudo-vector theory can be disposed of in a similar way, so there is no longer any *a priori* reason for preferring pseudo-scalar meson theory. Moreover, one is free to investigate the physical consequences of the pseudo-vector theory without resorting to any arbitrary cut-off procedure.

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ON PION-PAIR PRODUCTION IN NUCLEAR EMULSIONS

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In connection with a recent observation by Yagoda [1]) of a pion-pair in a nuclear emulsion, it seems worth while to recall that an event very similar to the one described in ref. [1]) has been observed by the author [2,3]). The event was found in an Ilford G—5 emulsion exposed at 2867 m (Pic du Midi). The pair originates from a point well inside the emulsion. One of the pions stops in the emulsion after $\approx 540\ \mu$ giving rise to what seems to be a σ-star consisting of a single low-energy proton track. The other one escapes from the emulsion after $\approx 2100\ \mu$. The variation of ionisation and scattering along the two tracks shows that they move in opposite directions. The angle of the fork is 152°. Mass-measurements were performed by ionization-scattering and range-scattering and yielded values of $(271 \pm 79)\ m_e$ and $(260 \pm 40)\ m_e$ for the two particles. There is no visible recoil. The kinetic energies of the two particles turn out to be (15.8 ± 2.0) MeV and (5.0 ± 0.6) MeV.

Elementary collision dynamics make highly improbable the production of this pair in a neutron-nucleus collision by either "plural" or "multiple" mechanism. Pair production by a photon is ruled out by the low total momentum of the two pions, requiring for the recoil nucleus a momentum noticeably greater than ≈ 200 MeV/c.

If, on the other hand, we consider the two pions as the decay products of the two-body decay of a V^0-particle, its mass turns out to be $(595 \pm 36)\ m_e$, which is within error limits the same as the value found by Yagoda [1]).

The emulsion has been searched carefully along the line of flight of the supposed V^0-particle for any event in which it might have been created. None were found in a distance of $\approx 500\ \mu$. Considering

† On leave from the Institute of Physics, Academy of the Rumanian People's Republic, Bucharest.

the low velocity ($\beta \approx 0.12$) of the V^0, this yields a value $\tau_0 \geqq 10^{-12}$ sec for its mean life-time. Thus, the possibility of a ζ^0-particle [4]) is also ruled out.

The possibility has also to be considered † that the two observed events are the result of the three-body decay of a hypothetical neutral τ-meson. As has been pointed out by Dalitz ††, if the τ^0 decays according to the scheme

$$\tau^0 \rightarrow \pi^+ + \pi^- + \pi^0, \tag{1}$$

the apparent Q-value of an assumed two-body decay may take any value between 0 and 80 MeV; one can easily show that

$$X = (M-\mu)\left[1 - \frac{2MT_0}{(M-\mu)^2}\right]^{\frac{1}{2}}, \tag{2}$$

where M and μ are the masses of the τ and π mesons, X the apparent mass of the assumed V^0, and T_0 the kinetic energy of the π^0 in the rest frame of the τ^0.

The two observed values of X lie very near to one another and most of the observed "anomalous" θ^0-decays exhibit when analyzed according to a $\pi^+ + \pi^-$-scheme [5]) a grouping around $X \approx 600\, m_e$ and a transverse momentum distribution characteristic for the two-body decay of a homogeneous group of particles. These facts would imply that the energy spectrum of the π^0's from τ^0-decay should be peaked somewhere around ≈ 33 MeV. But then, this feature should be visible also on the decay pattern of the charged τ-mesons coming to rest in nuclear emulsions.

By means of eq. (2) we have calculated the apparent X-values for all pairs of secondaries of 34 τ-mesons ††† listed in the "Report of the Committee on τ-Mesons" at the Padua Conference [6]). The frequency distribution of these X-values is plotted in Fig. 1. The average value of X is $\approx 616 m_e$. If the decay processes of τ^0 and τ^+ mesons are similar in all respects, then from Fig. 1 one would indeed expect most of the X-values from τ^0-decays to lie in the vicinity of $\approx 600 m_e$.

While the τ^0-meson hypothesis seems to solve the "phenomenological" side of the problem of "anomalous" θ^0-decays, it still offers no explanation for this peaking of the pion energy spectrum in

† I am indebted to Dr. B. Hyams for drawing my attention to this point.
†† Private communication from R. Dalitz to H. Yagoda, quoted in ref. [1]).
††† Only in 34 of the 40 reported cases were all the three pion energies measured.

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OPENING OF THE TAIT INSTITUTE OF MATHEMATICAL PHYSICS AT EDINBURGH (15—17 December 1955)

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Since the beginning of this academic year the Department of Mathematical Physics of Edinburgh University, of which Professor N. Kemmer, F.R.S.† (Tait Professor of Natural Philosophy) is the head, has occupied its own building at 1 Roxburgh Street, across the road from the Department of Natural Philosophy (Physics), headed by Professor N. Feather, F.R.S. On December 15th the building was formally opened by Sir Edward Appleton, F.R.S., Principal of Edinburgh University, and named the 'Tait Institute' in honour of P. G. Tait†† who was Professor of Natural Philosophy of Edinburgh from 1860 to 1901. To mark the occasion three open lectures of general scientific interest were delivered:

"Electronic Currents in the Ionosphere" — by Sir Edward Appleton, G.B.E., K.C.B., F.R.S.,

"On Forecasting the Future" — by Professor Sir Charles Darwin, K.B.E., F.R.S., Tait Professor of Natural Philosophy from 1923—1936,

"Problems of High Energy Physics" — by Professor R. E. Peierls, C.B.E., F.R.S. of Birmingham University.

It was hoped that Professor Max Born, F.R.S., Tait Professor of Natural Philosophy from 1936 to 1953, would also deliver a lecture in this series. Unfortunately he was unable to attend.

Following these lectures there was an informal conference of British mathematical physicists, attended by about fifty guests from other universities, at which topics of current interest in nuclear theory, high energy physics, and field theory were discussed. A small group also discussed questions in classical electromagnetic theory. Each session was headed by one or several introductory talks after which a considerable proportion of the time available was spent in quite informal discussion.

† Added in the proof (L.R.).

†† The development of Mathematical Physics at Edinburgh owes much to Tait's interest in the mathematical side of physics, and when in 1923 an independent chair in the subject was established, it was named after him.

τ-decays. If further research and better statistics will confirm this effect, its simplest explanation seems still to be the hypothesis advanced by Baker and Bergmann [7]) that (at least some of the) τ-mesons decay through an intermediate link, i.e. an unstable particle of mass $\approx 600 m_e$ which undergoes fast two-body decay into two

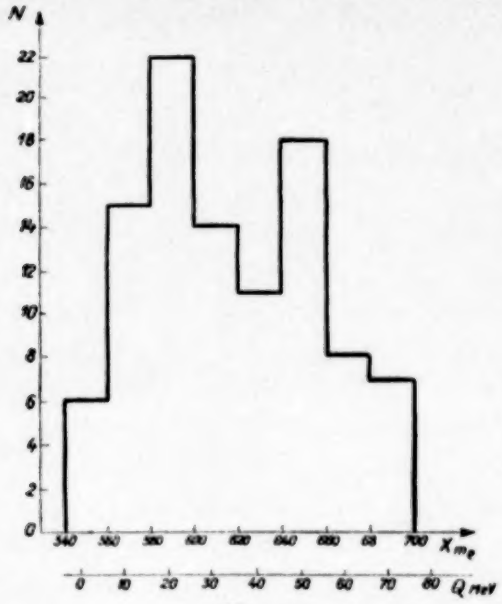

Fig. 1

pions. Of course, the cause of the peaking may also lie in the intrinsic mechanism of the direct three-body decay of the τ.

At any rate it seems highly indicated to gather better statistics as well on pion-pairs as on the energy spectrum of τ-meson secondaries.

The author extends his thanks to Professor M. Cosyns through the courtesy of whom he came in possession of the exposed and processed plates.

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The following are summaries of most of the main contributions. They are based on drafts supplied by the authors, but the responsibility for the form in which they appear rests with the editors (B. F. Bayman and P. W. Higgs).

1. General Field Theory

THE CONSISTENCY OF RENORMALIZED FIELD THEORIES

Dr. J. C. Taylor (Peterhouse, Cambridge) described an interesting approximation, suggested by Landau [1]) and his collaborators, for the divergent quantities in quantum electrodynamics. They neglect the fine structure constant, e^2, except where it occurs in the product e^2L (L is the logarithm of the cut-off). Their expression for the "bare" charge, e_B, has the remarkable property of becoming imaginary for sufficiently large L. While this result is suggestive, it of course proves nothing.

The results of the Russian authors can quickly be recovered by using Gell-Mann and Low's [2]) functional forms for the high energy propagators. But this method of derivation allows one to do more. By using the Landau limit ($e^2 \to 0$, $L \to \infty$, e^2L finite) as an intermediate step, the asymptotic value of e_B for large L but *finite* e^2 can be found, and is imaginary.

This theorem depends critically upon the existence of the Landau limit, which is assured if the functions are analytic in e^2 at $e^2 = 0$. Thus, if perturbation theory has a non-zero radius of convergence, it cannot provide a consistent solution of the electrodynamic equations. The same can be proved of renormalizable meson theories.

RECENT FORMULATIONS OF QUANTUM FIELD THEORY

In a short introductory report on recent formulations of quantum field theory Dr. J. G. Valatin (Birmingham) stressed two points:

(i) The functional field equations can be obtained by means of a direct quantization in which the primary quantities refer to an over-all space-time view. They form the basis of a formulation of quantum field theory which is independent of the state-vectors and space-like surfaces of the conventional approach.

(ii) Apparently different attempts to obtain equations for amplitudes and generating functionals are simply related and correspond to different representations in the same scheme.

For details see published papers [3,4]).

2. Meson Theory

THE LOW EQUATION FOR CHARGED SCALAR MESONS

Mr. L. Castillejo (University College, London) reported work done by R. H. Dalitz, F. J. Dyson and himself [5]) on the Low equations for the scattering of charged scalar mesons. These equations, in the one-meson, no-recoil approximation, are simple enough to make a general solution possible. The scattering amplitudes which are of the form $e^{i\delta} \sin \delta$, with real δ, and satisfy separate unitarity conditions are just the sum and difference of the amplitudes which satisfy dispersion relations [6]). This makes it possible to define a single function $H(\omega)$ of a complex variable ω which contains all the information provided by the Low equations, and has the following properties: (i) $H(\omega)$ is a meromorphic function in the ω plane cut from $-\infty$ to $-\mu$ and μ to ∞ along the real axis. (ii) Im $(H(\omega))$ has the same sign as Im (ω). (iii) Im $(H(\omega)) \to k/\sqrt{k^2+\mu^2}$ as $\omega \to \pm\sqrt{\mu^2+k^2}$ from the upper half plane, and $H(\omega^*) = H^*(\omega)$. (iv) $H(\omega)$ satisfies conditions at $\omega = 0$, $\omega = \infty$ and along the uncut part of the real axis which depend on the energies and effective coupling constants corresponding to the bound meson-proton states. As $\omega \to \pm\sqrt{\mu^2+k^2}$, Re $(H(\omega)) \to -k \cot \delta^{\pm}(k)/\sqrt{\mu^2+k^2}$ where $\delta^{\pm}(k)$ are the phase shifts for scattering of positive or negative mesons of momentum k on protons.

The complete solution for $H(\omega)$ follows by imposing condition (iii) on formulae giving the structure of functions satisfying (i) and (ii). $H(\omega)$ then consists of a particular integral, having the structure of a dispersion formula, plus an arbitrary Wigner R-function $R(\omega)$. Condition (iv) slightly restricts the possible values of the parameters in $R(\omega)$, but is quite insufficient to determine them uniquely. The Serber-Lee [7]) solution with $R(\omega) \equiv 0$ can only be obtained by imposing the artificial restriction that the scattering amplitudes tend to zero uniformly in k as the coupling constant tends to zero. The Low equation should therefore be treated, like a dispersion formula, as a restriction on the scattering, rather than as an equation from which the scattering can be determined.

SCATTERING OF CHARGED SCALAR MESONS

Dr. P. T. Matthews (Birmingham) reported on work done in collaboration with Gordon Feldman. Feynman's method of functional integration when applied to meson pair theory, with neglect of

nucleon recoil, gives very directly the exact solution to scattering. It shows clearly that this interaction, after renormalization, is equivalent to scattering of a Klein-Gordon particle in a δ-function potential. (This includes the effects of virtual meson pair creation.)

The same method applied to the ordinary charged scalar theory naturally suggests an approximation, which may be interpreted as the calculation of the scattering in an equivalent "potential", determined by Born approximation. The result differs from that of the Tamm-Dancoff approximation (which excludes virtual pair creation), but is identical with the "one meson" approximation of Chew and Low [8]), and the lowest order result of the variation principle of Cini and Fubini [9]).

AN ATTEMPT AT A GENERAL SOLUTION OF CHARGED SCALAR MESON THEORY

Dr. S. F. Edwards (Birmingham) told the conference how he has lately been setting up a new method of solution of field theory based on the evaluation of propagators in functional integration representation, e.g. how the scattering of mesons on nucleons can be derived from the meson-nucleon Green function [10])

$$\Delta_{\mathrm{MN}}(x_1 x_2, \xi_1 \xi_2) = N^{-1} \int \phi(\xi_1)\phi(\xi_2) G(x_1 x_2 \phi) \exp\{iA\} \delta\phi\delta\phi^*.$$

The method is that of finding the function ϕ, ϕ^* of stationary phase, and expanding the integrand, in the exponential, about this function, whereupon the integral can be evaluated.

In view of the interest in charged scalar theory at this conference, Dr. Edwards went on to describe the results of the method applied to this theory. There are two solutions to the function of stationary phase for all coupling constants, the one corresponding to the weak coupling solution, the other to the strong. When g^2 is small the value of the integral comes almost entirely from the weak coupling saddle function, and when g^2 is large from the strong coupling saddle function; for intermediate g^2 the solutions come close and the calculation, which in the limiting cases need only use linear and quadratic terms in ϕ, ϕ^*, has to be extended to cubic and quartic terms. The interesting point is that the region of transition from weak to strong coupling comes around $g^2/4\pi = \varkappa/d$ where d is the cut-off momentum. In the region $g^2/4\pi \ll \varkappa/d$ the solution for the scattering amplitude is that found by several authors, as described in Dr. Matthews' talk. When $g^2/4\pi \gg \varkappa/d$ the most striking feature is the appearance

of a stable isobaric state of the proton. If no cut-off is used, the strong coupling solution is always the correct one. The way g^2 and d appear in strong coupling is such that it does not appear possible in principle to renormalize the theory, except in the limit $g^2/4\pi \ll \varkappa/d$.

For further details, see ref. [10]).

ON GENERALIZED DISPERSION RELATIONS

Dr. Abdus Salam (St. John's College, Cambridge) showed how Goldberger's [11]) dispersion formula for forward scattering of mesons can be generalised to the non-forward scattering case. Let a meson of 4-momentum k be scattered by a nucleon of 4-momentum p, spin λ, the final nucleon state being p', λ'.

Choose a frame of reference in which $\mathbf{p}+\mathbf{p}' = 0$. The scattering amplitude M can be expressed, in this frame, as a function of k_0, the incident meson energy, and the variable

$$P = [\tfrac{1}{2}(p \cdot p' - \varkappa^2)]^{\frac{1}{2}}.$$

Holding P fixed, one obtains the dispersion relation

$$\operatorname{Re} M_{\lambda'\lambda}(k_0, P) = \frac{2}{\pi} P \cdot v \cdot \int_0^\infty \frac{k_0' \, dk_0' \operatorname{Im} M_{\lambda'\lambda}(k_0', P)}{k_0'^2 - k_0^2}.$$

In the c.m. frame, $P = k_c \sin \tfrac{1}{2}\theta_c$, where k_c is the magnitude of the 3-momentum.

It is clear, therefore, that on transformation to the c.m. system, one may isolate a phase-shift for an arbitrary l value on the left hand side. On the right hand side, however, there appears a complicated sum involving all phase shifts, at all energies. A number of approximation methods can be used to make effective use of this relation for the S and the D phase shifts.

PS-PV MESON THEORY

Dr. J. S. R. Chisholm (Cardiff) reported progress towards the renormalisation of pseudoscalar meson theory with derivative coupling. In neutral PS-PV meson theory, the Dyson transformation

$$\zeta = \exp(ig\gamma_5\phi)\psi$$

replaces the interaction Lagrangian by

$$\bar{\zeta}[\exp(ig\gamma_5\phi) - 1]\zeta M.$$

This exponential can be expanded, and a complex graphical system set up; a term

$$\bar{\zeta}(ig\gamma_5\phi)^n \zeta M$$

corresponds to a vertex with n meson lines and two fermion lines. The multiplicity of meson lines allows

a) many mesons lines joining every pair of vertices;

b) meson "loop lines", beginning and ending at the same vertex; these lines are eliminated by a renormalisation.

Graphs with only one line joining every pair of vertices are called "skeletons"; the S-matrix is the sum of terms corresponding to all skeletons, with new propagators representing propagation of any number of mesons. The multi-meson propagators are $d_e = \cosh \Delta_F$ (even parity) and $d_o = \sinh \Delta_F$ (odd parity); the fermion propagators are $s_e = S_F \cosh \Delta_F$ and $s_o = S_F \sinh \Delta_F$. The vertex factors are M or $M\gamma_5$, depending on the product of parities of the lines at the vertices. A further renormalisation eliminates vacuum-like divergences. Arnowitt and Deser [12]) have derived the same results by functional integration; they have suggested an analytic continuation method for avoiding essential singularities in the new propagators.

Note added later: Fermion line propagators can be iterated any number of times; the propagators s_e and s_o must be replaced by iterated propagators, which are easily calculated once we have found s_e and s_o in momentum space.

3. Elementary Particles

CLASSIFICATION OF ELEMENTARY PARTICLES

Dr. A. Salam reported on the attempts to formulate theories of elementary particles from a fundamental point of view. Historically, Pais end Gell-Mann [13]) first made an assignment of basic spins for the elementary particles ($I = \frac{1}{2}$ for nucleons and 0 mesons; these we shall call "iso-fermions"; $I = 0$ for Λ^0, $I = 1$ for π mesons and Σ hyperons). Strong interactions are those which conserve Q (the charge), and I_z. From the fundamental point of view, this would imply that the interaction Lagrangian is invariant for isobaric rotations.

On the analogy of the relation between charge and isobaric spin ($Q = I_z + \frac{1}{2}N$ where $N = +1$ for a fermion and -1 for an anti-fermion), one would like to have a general relation connecting I_z and Q. Gell-Mann suggested

$$Q = I_z + \tfrac{1}{2}N + \tfrac{1}{2}S,$$

this equation defining the "strangeness" number S.

With the above isobaric spin assignments, $S = 0$ for mesons and nucleons, -1 for $\bar{\theta}^0$, θ^-, Λ and Σ particles, and -2 for Ξ particles. The present experimental results could all be restated thus: strong interactions conserve Q and S. It might, however, be difficult to find a fundamental basis for the conservation of S; or alternatively, if one still postulates invariance for rotations in isobaric space, to find a basis for the relation $Q = I + \frac{1}{2}N + \frac{1}{2}S$.

Prentki and d'Espagnat [14]) noticed that in a strong interaction, not only the fermion number N is conserved, but also the number of iso-fermions (U). Thus both the initial and the final states in the reaction $\pi + N \rightarrow \Lambda + \theta$ contain one iso-fermion. One could in fact write $Q = I_z + \frac{1}{2}U$ (i.e. $U = S + N$) and state that strong interactions are those which conserve Q and U or Q and I_z.

On the fundamental side, Prentki and d'Espagnat made the amazing discovery that if, with the present list of elementary particles and their present assignments, the interaction Lagrangian is made invariant under isobaric rotations as well as isobaric reflections, not only is the conservation of I_z guaranteed, but also that of U: U acts as the isobaric parity quantum number. (The cascade particle Ξ needs special treatment; it has to be classed as an anti-fermion.) This scheme seems by far to be the most elegant so far devised.

A different scheme has been advanced by Salam and Polkinghorne [15]) using the ideas of [4] isobaric spin space. One writes $Q = I_z^+ + I_z^-$. This is the same relation as Pais postulated for bosons; it differs, however, from his relation for fermions. I_z^+ takes the place of the conventional [3] isobaric spin component; I_z^- is analogous to "strangeness".

The nucleons, the cascade particles, as well as the four θ particles correspond to the $(\frac{1}{2},\frac{1}{2})$ representation in the [4] space; the scalar (0,0) gives Λ and (1,0) the π mesons and Σ hyperons. The τ mesons may well correspond to the (0,1) representation, τ^+ thus being produced in association with Ξ rather than Λ or Σ, as in the Gell-Mann scheme. An attractive feature of the scheme is tnat all particles correspond to the single-valued representations of the isobaric group. However, the nucleon and cascade particle mass difference remains unexplained.

BINDING ENERGIES IN LIGHT HYPERNUCLEI

Dr. R. H. Dalitz (Birmingham) discussed the interpretation of binding energies B_Λ for the Λ_0-hyperon bound in hypernuclei $A \leqq 5$.

Comparison of the B_Λ for ${}_\Lambda H_4$ (1.3 ± 0.3 MeV) and ${}_\Lambda He_4$ (1.2 ± 0.5 MeV) indicates charge-symmetry in the Λ_0-nucleon interaction, the Λ_0 being self charge-symmetric, as is required in the Gell-Mann scheme. In consequence the interaction $\Lambda_0 \to \Lambda_0+\pi^0$ is forbidden and the Λ_0-nucleon interaction will arise from exchange of a K-meson or of several pions, and must therefore be of short range ($\lesssim \hbar/2\mu c$). The occurence of the decay ${}_\Lambda H_4 \to He_4+\pi^-$ implies that the spin of the mirror nuclei ${}_\Lambda H_4$, ${}_\Lambda He_4$ can only be $0-$, $1+$, $2-$, $3+$ . . . A determination of the spin of ${}_\Lambda H_4$ would therefore allow the parity of the Λ_0 to be deduced. Comparison of B_Λ for ${}_\Lambda He_5$ (2.8 ± 0.5 MeV) with that for ${}_\Lambda He_4$ shows that the He_4 core attracts the Λ_0-hyperon only a little more than does the He_3 core. If the Λ_0-nucleon interaction were spin-independent, a binding energy B_Λ of 6 MeV at least would be expected for ${}_\Lambda He_5$, taking into account the known parameters of He_4 and He_3: it is concluded that the Λ_0-nucleon interaction has a strong spin-dependence in the S-state. In ${}_\Lambda H_3$ ($B_\Lambda = 0.3\pm0.3$ MeV) the most favourable spin orientations of Λ_0, N and P are effective. For example, if the Λ_0 has spin $\frac{1}{2}+$, it would follow that the spin of ${}_\Lambda He_4$ is $1+$, that the parallel Λ_0-nucleon spin orientation is most favourable and that ${}_\Lambda H_3$ should have spin $\frac{3}{2}$. On this picture, the absence of ${}_\Lambda H_2$ follows naturally from the short range nature of the Λ_0-nucleon interaction, the absence of the $T = 1$ multiplet containing ${}_\Lambda N_3$ and ${}_\Lambda He_3$ is natural since the Pauli principle requires the less favourable spin orientation to be effective. Finally, Holloday's suggestion that Σ^- and neutrons may form compound systems was discussed briefly, the complex (Σ^-NN) being considered the most favourable possibility if there is an attractive neutron force.

THE RELATIONSHIP OF τ AND θ MESONS

If τ and θ mesons have the same spin and parity, Dr. Dalitz proceeded, the following configurations: (a) emission of low energy negative pions, (b) emission of negative pions of energy near to the maximum possible pion energy, (c) decays in which the pions are approximately collinear, are expected to be strongly unfavoured, relative to the weight of each configuration in phase space. In the present total of ≈ 300 τ-decay events observed by the Columbia, Bristol, M.I.T. and Berkeley groups from Bevatron exposures, the relative frequency of these configurations is about that expected from their weight in phase space, and it may therefore be concluded that the τ and θ mesons do not have the same spin and parity.

The simplest spin possibilities then allowed for the τ-meson are 0− and 1+. To compare these, the data are analysed by calculation of $N = \Sigma_i \ln(P_i)$, summed over all events, P_i being the probability density for a spin 1+ decay (explicitly $P = E/E_{max}$), evaluated for the i-th event. The Columbia data (63 events) give $N = -7.7$, i.e. odds $10^{7.7}$ to 1 against spin 1+ compared with spin 0−. For isotropic decay the expected value of N is -5.6 ± 2.5 for 63 events, the standard deviation corresponding to random sampling. However if P above really represents the decay i.e. the spin is 1+, the expected value of N is 3.8 ± 1.6. The present M.I.T. data (54 events) include rather few slow negative pions and lead to $N = 0.8$, about midway between the expected values for 0− and 1+. However, taken together, these 117 events give $N = -6.9$, to be compared with expected values of -10.4 ± 3.4 for 0− and 7.1 ± 2.1 for 1+. It therefore appears rather unlikely that the τ-meson is pseudovector. This conclusion is given further support by the Bristol and Berkeley data. Higher spin values (2−, 3+, 4− . . .) may still not be incompatible with the present data.

Dr. Dalitz then summarized the experimental evidence on K-meson masses and lifetimes, and discussed the possibility of a parent-daughter relationship between τ and θ mesons, as suggested by Orear and Lee.

4. Nuclear Theory

THE BRUECKNER THEORY OF NUCLEAR STRUCTURE

A discussion of the Brueckner theory of nuclear structure occupied one and a half sessions of the conference. The principal speaker at these sessions was Dr. R. J. Eden (Manchester) who devoted his first lecture to a general account of the theory and in his second lecture indicated a number of possible applications of the method. The object of the Brueckner theory is to explain certain aspects of nuclear structure from a knowledge of the forces which act between nucleons in free space. It assumes that only two-body forces between nucleons are important in determining nuclear properties but allows for the essential modification to these forces due to the operation of the exclusion principle. The starting point of the theory is the many-body Schrödinger equation to which a formal solution can be constructed in terms of the wave function for a nuclear model. Provided that certain correction terms can be neglected this nuclear model is simple and resembles the nuclear shell model. The wave function of the model has to be determined from a set of self-consistent equa-

tions which represent a major advance on the Hartree equations in that they can be applied to a system having strong short range forces of the type encountered in the nucleus. The solution of the Brueckner self-consistent equations represents a formidable task, but the problem simplifies when a very large nucleus (infinite nuclear medium) is considered. Dr. Eden indicated some recent results of H. A. Bethe on this simplified problem both for repulsive core and for Yukawa type potentials between nucleons.

In his second talk Dr. Eden indicated an extensive range of applications of the method which included the derivation of the shell model potential (including spin-orbit coupling), a study of the shape of the nuclear edge, the derivation of the optical model potential and the relation of the method to nuclear resonances, and an extension of a method due to R. E. Peierls for studying collective motion in the nucleus. A preliminary consideration of some of these applications has already been made by one or more of the following: K. A. Brueckner, N. C. Francis, L. Kisslinger, C. A. Levinson, K. M. Watson and the speaker.

THE BASIS OF THE COLLECTIVE MODEL OF NUCLEAR MOTION

Professor R. E. Peierls and Mr. J. Yoccoz (Birmingham) have been trying to set up a derivation of the collective model of A. Bohr from the basic quantum equations. The idea is to start from the variation principle with the complete nuclear Hamiltonian, using as a trial function a linear superposition of Slater determinants, each built from the eigenstates for particles in a non-spherical well of suitable orientation. The weight factor with which these functions are combined is then a function of the orientation, and to make the energy stationary this has to be chosen as a spherical harmonic, if the well has axial symmetry, or as the wave function of an asymmetric top, in the more general case.

This method makes rotational variables appear in the wave function naturally, while still retaining a description in terms of shell model type wave functions. The energy comes out to sufficient approximation as a rotational energy and the method provides a means of estimating the effective moment of inertia. Preliminary estimates based on an harmonic oscillator well, which is the most convenient for calculations, indicate that the order of magnitude of the moment of inertia is reasonable.

The same principle can be applied to describe the translational

motion of a nucleus, starting from shell model wave functions, and thus to generalize to an arbitrary shape of well the results of Elliott and Skyrme [16]) for an oscillator well. However, it is not obvious that this gives the translational energy correctly as a function of momentum, and if this answer did not come out right it would throw doubt on the estimates of the rotational energy obtained in the first case. Professor Peierls and Mr. Yoccoz are at present attempting to clarify this apparent difficulty.

A SYSTEMATIC VARIATIONAL APPROACH TO NUCLEAR STRUCTURE

Arising out of the work of Elliott and Skyrme, emphasising the desirability of eliminating the centre of gravity motion in the nuclear shell model, Professor Jahn (Southampton) described a systematic method for constructing nuclear orbital states out of internal vectors. These internal vectors are taken to be (A is the nuclear mass number)

$$\mathbf{t}_a = \frac{1}{\sqrt{a(a+1)}}(\boldsymbol{\rho}_1+\boldsymbol{\rho}_2+\ldots+\boldsymbol{\rho}_a-a\boldsymbol{\rho}_{a+1}) \qquad (a = 1, 2, \ldots, A-1),$$

with $\boldsymbol{\rho}_i = \mathbf{r}_i/a_0$, where $\mathbf{r}_i$ ($i = 1, 2, \ldots, A$) is the vector drawn from the well-centre to particle i and a_0 is some convenient length. The vectors $\mathbf{t}_a$ transform under permutations of the particle numbers according to the orthogonal irreducible representation $[A-1,1]$ of the symmetric group S_A. Choosing a complete set of trial functions

$$\phi_{Nlm}(\mathbf{t}_a) = u_{Nl}(t_a)Y^l_m(\theta_a, \phi_a)$$

in the space of each individual vector $\mathbf{t}_a$, orbital trial functions of definite total angular momentum and definite permutation symmetry are formed by vector-coupling and application of appropriate Young operators. This leads to the construction of new orbital fractional parentage coefficient tables, which may be combined with already published $(N;N-2,2)$ charge-spin fractional parentage coefficient tables to give totally antisymmetric wave functions with the centre of gravity motion removed and the motion of the last pair of particles separated. The method leads also to a simplified method of calculating the energy matrix, with both central and tensor forces, reduction being made, by transformation to centre of gravity and internal motion for the last pair of particles, to a single radial integral over the internal radial motion for the last pair of particles and the distance function of the interaction (cf. Talmi). The aim is to

reach convergence, so that the addition of more states or the introduction of more variational parameters into the individual radial states makes no further change in the energy levels calculated with a given interaction.

THE STRUCTURE OF NUCLEI AROUND O^{16}

The programme of intermediate coupling calculations being carried out for the individual particle model of the nuclei around O^{16} by B. H. Flowers and J. P. Elliott at Harwell was described by the latter author.

The results for the lowest configurations of mass 18 and 19 lead to the following general conclusions [17]:

(i) The mixing of the (1d) and (2s) shells is large.

(ii) The mode of coupling for F^{19} is near the L-S limit.

(iii) The relative positions of the low levels of F^{19} are insensitive to the exchange character of the central force and also to the inclusion of a tensor force. This cannot be said for the levels of O^{19}.

(iv) The introduction of weak surface coupling of the Bohr-Mottelson type necessary to explain the 873 keV E2 transition in O^{17} gives simultaneous agreement with the 197 keV E2 transition in F^{19} without noticeably shifting the levels.

(v) From the O^{18}—F^{18} and O^{19}—F^{19} energy differences one can deduce the approximate values for the exchange parameters of the central force:

$$W+M = -0.85$$
$$W+B = -0.37$$
$$\text{with} \quad W+M+H+B = -1.$$

The disturbing feature of the F^{19} spectrum is the occurrence of an odd parity level, which must arise from an excited configuration, at 110 keV above the ground state. To try and understand this a series of calculations for the simplest excited configurations have been started, namely for the odd parity levels of O^{16}, N^{15} and O^{17}. Results for O^{16} and N^{15} are very encouraging if one takes into account the motion of the centre of mass in the shell model [16].

In the calculations mentioned so far no account has been taken of the tensor force. It has been shown [18]) that the inclusion of a tensor force for mass 14 explains the long β-lifetime of C^{14} and causes very little change in the good agreement for the spectrum found without such a force. It is being investigated in which other nuclei this tensor force may be important.

COLLECTIVE MOTIONS OF A SHELL STRUCTURE

Professor L. Rosenfeld (Manchester) reported upon recent attempts, following a general suggestion of Inglis [19]), at establishing a connexion between the shell structure of a nucleus and its collective motions [20]). The underlying idea is that if the nucleons are undergoing some kind of collective motion of definite frequency, the potential representing the average interaction of any one of them with the others will oscillate with the same frequency. If, therefore, one considers the individual states of the nucleons in an oscillating potential, and writes down the corresponding expectation value of the total energy of the system, the condition that this expectation value be stationary will be expected to determine the proper frequencies of the collective modes in terms of the parameters characterizing the shell structure.

It was stressed that the method is still lacking a completely satisfactory justification, while it has already given some promising results and is susceptible of a very general formulation without excessive complication.

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BOOK REVIEWS

M. E. Rose, *Multipole fields* (John Wiley & Sons, New York and Chapman and Hall, London 1955. viii-99 pages. £ 2).

This is not, unfortunately, the much needed monograph on multipole fields and their applications, which the title and the distinguished author's name would lead one to expect. The author has rendered a great service to nuclear studies by devoting considerable effort and care to developing this subject and its various aspects; it cannot be said that he has devoted much care to the composition of this book — if, indeed, he has not just "composed" it by collecting stray notes from his drawers and sending them straight to the printer.

Neither the preface not a careful reading of the book has made it clear to me exactly the author set out to do. There is nowhere any clear statement of the physical problems concerned or of the methods for subjecting them to mathematical analysis. It seems that the author wanted to present elaborate calculations of fields, intensities, matrix elements and the like, performed by means of a uniform technique; but he has not attempted either any exhaustive, systematic discussion of this mathematical technique. The book therefore, is clearly of no use to the student. As to those physicists who already know all about the subject, they will find in it a compendium of the basic definitions and formulae pertaining to multipole fields, a summary exposition of their main properties and, filling the latter half of the small volume, a derivation of the formulae governing internal conversion and multipole emission.

The author's mode of exposition is not felicitous; all too often, excessive brevity leads to obscurity, and no sufficient attention is paid to the order in which the parts of an argument should be presented to make their significance clear from the start, without forcing the reader to retrace his steps repeatedly. The uncouth "*Physical Review* dialect adopted by the author does not improve matters. The treatment of internal conversion and emission, in particular, is completely topsy-turvy: the reader is constantly referred to results which are going to be derived in the next stage of the argument, until he begins to wonder whether his own brain, or the author's, has undergone a time-inversion.

Yet, there is plenty of good stuff in this small, but extremely competent treatise: many nice points are made — about pitfalls in the choice of a special gauge for the potential (sec. 26), the essentials of angular correlation, alignment and orientation problems (sec. 32), etc. — which are both inetresting and useful. It is all the more a pity that they should be presented in such an uninviting fashion.

The list of references is proudly described as "complete"; and, indeed, it starts promisingly enough with Abraham (Max). But, believe it or not, the name of Rubinowicz does not occur in it.

The book is neatly printed, although the omission of punctuation marks after displayed formulae is a typographical mannerism which has nothing to commend it. The price is too high.

L. R.

G. P. Harnwell and W. E. Stephens, *Atomic physics: an atomic description of physical phenomena* (McGraw-Hill Book Co, New York, Toronto, London 1955. x-401 pages. 8, α 3).

This textbook, whose subject is outside the scope of the journal, can only be briefly reviewed. After two introductory chapters (93 pages), there is an elaborate

treatment of atomic and molecular spectroscopy in 3 chapters (186 pages), including the quantum theory of radiation and hyperfine structure. The last two chapters (108 pages) discuss the properties of matter in bulk, or more accurately the influence of the quantal features of the elementary processes on such properties. The book is thus meant as a complement to some more elementary exposition of the atomic theory of matter. The treatment is theoretical, but with the emphasis on the physical rather than the mathematical features. There are numerous interesting problems and good references for further study at the end of each chapter.

L. R.

R. S. SHANKLAND, *Atomic and nuclear physics.* (The MacMillan Company, New York 1955. xv-529 pages. 44 s.)

This textbook provides a sound general introduction to a vast subject. The material is set out in a clear and concise manner and by concentrating on essentials the author has succeeded in compressing a considerable amount of information within a book of manageable proportions. In addition, he has found space to describe recent important developments in such fields as the fine structure of the hydrogen spectrum lines and the production of the transuranic elements to quote but two examples. Where the treatment is more superficial as for example in the chapter on the solid state of matter, adequate references to the original papers on the subject are given. The book is well indexed and is supplemented by a comprehensive list of physical constants.

G. N. FOWLER

FOR PROSPECTIVE CONTRIBUTORS

Since the nomenclature in the field of Nuclear Physics is not yet quite unified and a number of illogical expressions are unfortunately still widely used, it is advisable to give the following indications about the terminology adopted in this journal.

(1) The word 'electron' applies to particles with an elementary charge of either sign. When it is desirable to emphasize the sign of the charge, the words 'positon' and 'negaton' are used. Thus the illogical phrase 'electron-positron pair' is replaced by 'electron pair', or, if required, by 'positon-negaton pair'.

(2) Nuclei (or nucleids, *not* 'nuclids') are conveniently divided according to the parity of the proton and neutron numbers Z, N, into four families as follows:

even mass nuclei	even nuclei (Z even, N even) odd nuclei (Z odd, N odd)
odd mass nuclei	odd proton nuclei (Z odd, N even) odd neutron nuclei (Z even, N odd).

(3) The expressions 'isotopic spin', 'isotopic quantum number' are replaced by 'isobaric spin', 'isobaric quantum number'.

(4) For the elementary particles use is made of the latest nomenclature adopted by international agreement.

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